

Moving a Well Away from a River Delays River Capture as the Square of Distance, with a Long Tail Near the End

— the time for the river share to go from half to 95 per cent is far longer when the aquifer is seen as an extended body with distance than as a single box, and the remainder falls only as the square root of time —the separator is whether the aquifer is seen as a single box or as an extended body with distance — [Paper 956]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

September 2026

YOI: <https://www.yamagishiyuuki.com/yoi/956>

— *From thinking to syncing.* —

Abstract

When groundwater is pumped from a well near a river, at first water stored in the aquifer comes out, but in time much of the pumped water is drawn from the river. This paper finds, by formula and by numerical integration with an image well, how the share of pumped water coming from the river grows with time once the distance between well and river is included, and compares it with the single-box model of Paper 937. No new theorem or law is claimed. First, the formula and the image-well integration agreed: the share from the river was 0.025347319, 0.479500122 and 0.943628022 at dimensionless times 0.1, 1 and 100, with formula and numerical integration agreeing within 10^{-8} (Section 2). Second, from half to 95 per cent takes far longer than in the box model: the share reaches 0.5 at dimensionless time 1.099055 and 0.95 at 127.157222, a ratio of **115.696904**. In the single-box model of Paper 937 this ratio was 4.321928 (Section 3). Third, the remainder falls only as the square root of time: the share not from the river was 0.056372 at dimensionless time 100 (0.056419 from one over the square root of pi times time) and 0.005642 at 10000 (0.005642) (Section 3). Fourth, moving the well twice as far from the river makes the time to reach the same share four times as long: setting transmissivity 500 m²/day and storativity 0.2, the share reaches half in 4.396 days at 100 m and 17.585 days at 200 m, and 95 per cent in 508.629 and **2034.516** days (Section 3). Fifth, the separator is whether the aquifer is seen as a single box or as an extended body with distance: as a box, capture from the river proceeds with a single time constant. As an extended body, time scales with the square of distance, and a long tail remains in which the remainder falls only as the square root of time (Section 4). **Keywords:** stream depletion, groundwater pumping, Glover-Balmer formula, image well, aquifer, complementary error function.

Scope of this paper (scope note): No new theorem or law is claimed: the formula for the share from the river for a well near a river (Glover and Balmer), the Theis equation, and representing a river with an image well are all known in hydrogeology. Only ideal conditions are treated: the aquifer is uniform and unbounded, the river fully penetrates it with constant level, and there is no streambed resistance. Shallow rivers, aquifer boundaries and recharge from rain are not included. These are set values: transmissivity 500 m²/day, storativity 0.2 and distances 50, 100 and 200 m are set for comparison and correspond to no aquifer. How much to pump is not said: pumping amounts that keep river flow are not treated. Relation to earlier papers: Paper 937, in its scope note, did not include space, treating the aquifer as a single box and leaving out the distance between well and river and the spread of water levels; this paper sits in that seat (no earlier occupant). Paper 937 showed that pumping up to the recharge stops groundwater decline, but only when nearly all the water once flowing to the river has been taken. Here it is counted that with the distance between well and river, the time to reach that “nearly all” has a far longer tail than a single box. Paper 599 showed that around a well the water table falls only as the logarithm of distance, and keeps falling as the logarithm of time: the image well here superposes two such Theis drawdowns. What is added: checking the formula by numerical integration with an image well, comparing the ratio of times to half and to 95 per cent with the box model, showing numerically that the remainder falls as the square root of time, giving days for each distance with set values, and placing the separator on “a single box, or an extended body with distance”.

1. Notation and the Question

Setting: a well at distance d from a river pumps at rate Q . The aquifer has transmissivity T and storativity S . Treating the river as a line of constant level, an injection image well at the same distance on the far side keeps the level on the river line unchanged. The share of pumped water coming from the river is (Glover and Balmer):

$$\frac{q_{\text{river}}}{Q} = \operatorname{erfc}\left(\sqrt{\frac{d^2 S}{4Tt}}\right) = \operatorname{erfc}\left(\frac{1}{2\sqrt{t_D}}\right), \quad t_D = \frac{Tt}{Sd^2}$$

Check: the flow across the river line is found from the gradients of the Theis drawdowns of the two wells and integrated numerically along the river.

Model for comparison: in the single-box model of Paper 937, the share from the river is $1 - e^{-t/\tau}$.

Question: including the distance between well and river, how fast is pumped water replaced by water from the river?

2. Formula and Numerical Integration

Computation 2.1: share from the river.

dimensionless time t_D	formula	image-well numerical integration
0.1	0.025347319	0.025347319
0.5	0.317310508	0.317310508
1	0.479500122	0.479500122
2	0.617075077	0.617075077
10	0.823063274	0.823063274
100	0.943628022	0.943628022

At six times, formula and numerical integration agreed within 10^{-8} .

3. Time to Reach and the Tail

Computation 3.1: time for the share to reach 0.5 and 0.95.

model	reaches 0.5	reaches 0.95	ratio
extended body (t_D)	1.099055	127.157222	115.696904
single box (multiples of τ)	0.693147	2.995732	4.321928

Computation 3.2: the remainder not from the river, $1 - q_{\text{river}}/Q$, against $1/\sqrt{\pi t_D}$: 0.056372 against 0.056419 at $t_D = 100$, 0.017840 against 0.017841 at 1000, and 0.005642 against 0.005642 at 10000.

Computation 3.3: days with transmissivity 500 m²/day and storativity 0.2.

well-river distance	days to 0.5	days to 0.95
50 m	1.099	127.157
100 m	4.396	508.629
200 m	17.585	2034.516

Doubling the distance multiplied the days by 4.000000.

Observation 3.4

half is reached quickly, but a long tail follows. With an extended body, the share took 115.696904 times as long to go from 0.5 to 0.95, far longer than the 4.321928 times of a single box: the remainder followed $1/\sqrt{\pi t_D}$, falling only as the square root of time, and at a distance of 200 m reaching 95 per cent took 2034.516 days.

4. The Separator

Observation 4.1

the separator is whether the aquifer is seen as a single box or as an extended body with distance. In a single box, stored water declines with one time constant and capture from the river rises in step with it: in an extended body, the remainder fell only as the square root of time (Computation 3.2). Time scales with the dimensionless time $t_D = Tt/(Sd^2)$, so doubling the distance made the time to reach the same share four times as long.

Observation 4.2

what is not said. Real aquifers and rivers, streambed resistance, shallow rivers, aquifer boundaries, recharge, and how much to pump are not treated. What is shown is that under ideal conditions, including the distance between well and river gives the share from the river a long tail that lengthens as the square of distance.

5. Roll Call and Closing

1. Glover-Balmer formula and image well (Section 1): dimensionless time t_D .
2. formula and numerical integration agree (Computation 2.1): within 10^{-8} .
3. 115.696904 times from half to 95 per cent (Computation 3.1): 4.321928 times in a box.
4. remainder $1/\sqrt{\pi t_D}$ (Computation 3.2): falls only as the square root of time.
5. 2034.516 days at 200 m (Computation 3.3): twice the distance, four times the time.
6. a single box, or an extended body (Observation 4.1): the separator; the scope note of Paper 937; Paper 599.

Closing: Moving a well away from a river delays river capture as the square of distance, with a long tail near the end. The share from the river agreed between formula and image-well integration, and going from half to 95 per cent took 115.696904 times as long: seen as a single box this ratio is 4.321928, and the tail in which the remainder falls only as the square root of time appeared only once distance was included. With set values, reaching 95 per cent at 200 m took 2034.516 days. The separator is whether the aquifer is seen as a single box or as an extended body with distance. Placed among the earlier papers: the time to reach the “nearly all” that Paper 937 showed with a single box was counted with the distance Paper 937 left out, using the Theis drawdown of Paper 599 as an image well. To be honest: real rivers and aquifers depart from ideal conditions. How including distance changes the shape of the approach, and no further.

References

1. Glover, R. E., & Balmer, G. G. (1954). River depletion resulting from pumping a well near a river. *Transactions, American Geophysical Union*.
 2. Theis, C. V. (1935). The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage. *Transactions, American Geophysical Union*.
 3. Yamagishi, Y. (2026). *Around a Well the Water Table Falls Only as the Logarithm of Distance, and Keeps Falling as the Logarithm of Time* [Paper 599]. YOI: <https://www.yamagishiyuuki.com/yoi/599>; DOI: 10.5281/zenodo.23014427
 4. Yamagishi, Y. (2026). *Pumping Up to the Recharge Stops Groundwater Decline, but It Stops Only When Nearly All the Water Once Flowing to the River Has Been Taken* [Paper 937]. YOI: <https://www.yamagishiyuuki.com/yoi/937>
-

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)