

A Wound String Carries Its Weight in the Winding, and Its Overtones Stray Far Less Than a Plain String of the Same Weight

— tuned to the same pitch the tension is the same, but if only the thin core has bending stiffness the stiffness coefficient drops to the fourth power of the ratio of core to plain-string diameter —the separator is whether weight and bending stiffness are carried by one cross-section or split between separate parts — [Paper 958]

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Abstract

A low string must be heavy, but thickening a plain steel string also increases its bending stiffness, pushing overtones sharp of whole multiples (Paper 942). This paper tunes a string with copper wire wound on a steel core and a plain steel string of the same linear density to the same pitch, and compares their stiffness coefficients and overtone departures. No new theorem or law is claimed. First, with the same linear density, the tension at the same pitch is the same: the linear density 5.629122×10^{-3} kg/m of a string with a 0.4 mm core and 0.3 mm winding equals that of a plain steel string 0.955522 mm in diameter, and at a length of 0.648 m and 110 Hz both carry a tension of 114.403 N (Section 2). Second, most of the weight is carried by the winding: the winding's share of the linear density was 0.824758 (Section 2). Third, the stiffness coefficient drops to the fourth power of the ratio of core to plain-string diameter: the coefficient was 5.163611×10^{-5} for the wound string and 1.681419×10^{-3} for the plain string, a ratio of **0.030710** (Section 3). Fourth, the overtone departures are far smaller: the 20th tone departed by **17.6520** cents on the wound string and **443.7844** cents on the plain string, **25.140729** times as much. The wound-string formula matched finite-difference eigenvalues to a relative difference below 10^{-4} (Section 3). Fifth, the separator is whether weight and bending stiffness are carried by one cross-section or split between separate parts: in a plain string, adding weight adds bending stiffness with it. A wound string gives the weight to the winding and the bending stiffness to a thin core, so the overtone departures can be made small while keeping the weight that sets the pitch (Section 4). **Keywords:** wound string, inharmonicity, bending stiffness, linear density, overtones, string vibration.

Scope of this paper (scope note): No new theorem or law is claimed: the overtone formula for a string with bending stiffness, that wound strings are a device for making low notes with a thin core, and that the winding contributes little to bending stiffness are all known in the physics of musical instruments. Winding stiffness is set to zero: the winding adds only weight, and only the core has bending stiffness. The small real contribution of the winding is not included. These are set values: the length 0.648 m is as in Papers 857 and 942; the 0.4 mm core, 0.3 mm winding, copper density 8960 kg/m³ and pitch 110 Hz are set here. They correspond to no instrument string. Instruments and tuning are not treated: real guitars and pianos, tuning that widens intervals to match the departures, and how the tone is heard are not treated. Relation to earlier papers: Paper 942, in its scope note, did not treat instruments and tuning, leaving out wound strings, real pianos and guitars, tuning that widens intervals to match overtone departures, and how the tone is heard; this paper sits in the seat of wound strings (no earlier occupant). Paper 857, in its scope note, also did not include wound strings, leaving out strings with wire wound on a core; this paper sits in that seat as well. Paper 942 showed that bending stiffness sharpens overtones past whole multiples, and at the same pitch a thicker string strays further. Here it is counted how much smaller those departures become when the bending stiffness alone is confined to the core while the weight stays the same. What is added: showing that a wound string and a plain string of the same linear density carry the same tension, deriving the stiffness-coefficient ratio as the fourth power of the ratio of core to plain-string diameter, setting out the overtone departures, checking the formula against finite-difference eigenvalues, and placing the separator on “one cross-section, or separate parts”.

1. Notation and the Question

Setting: length $L = 0.648$ m (as in Papers 857 and 942), pitch $f_0 = 110$ Hz. Steel density 7850 kg/m³, copper 8960 kg/m³, Young’s modulus 200 GPa. The wound string has copper wire of diameter $d_w = 0.3$ mm wound without gaps on a core of diameter $d_c = 0.4$ mm, and its linear density is the sum of core and winding.

$$\mu = \rho_s \frac{\pi d_c^2}{4} + \rho_{Cu} \frac{\pi^2 (d_c + d_w) d_w}{4}$$

Let d_{eq} be the diameter of a plain steel string with the same linear density. Tuned to the same pitch, the tension $T = \mu(2Lf_0)^2$ is the same for both, and the stiffness coefficient is set by the diameter carrying bending stiffness (d_c for the wound string, d_{eq} for the plain string).

$$B = \frac{\pi^3 E d^4}{64 T L^2}, \quad \frac{B_{\text{wound}}}{B_{\text{plain}}} = \left(\frac{d_c}{d_{eq}} \right)^4$$

Overtone departures are counted in cents, $1200 \log_2(f_n/(nf_1))$, as in Paper 942.

Question: for strings of the same weight, how much smaller do overtone departures become when only the core carries bending stiffness?

2. Linear Density and Tension

Computation 2.1: linear density: core 9.864601×10^{-4} , winding 4.642662×10^{-3} , total 5.629122×10^{-3} kg/m (winding share 0.824758). The plain steel string of the same linear density has diameter 0.955522 mm. Both carry a tension of 114.403 N.

Observation 2.2

the winding carries most of the weight, and tension is set by weight alone. With the winding carrying 0.824758 of the linear density, the core can stay thin at 0.4 mm: with the same linear density, length and pitch, the tension is the same 114.403 N, and the two strings cannot be told apart by tension.

3. Stiffness Coefficient and Overtone Departures

Computation 3.1: the stiffness coefficient was 5.163611×10^{-5} for the wound string and 1.681419×10^{-3} for the plain string; the ratio **0.030710** matched the fourth power of core over plain diameter, 0.030710, within 10^{-12} .

Computation 3.2: overtone departures (cents).

string	$n = 2$	$n = 5$	$n = 10$	$n = 20$
wound	0.1341	1.0720	4.4135	17.6520
plain	4.3481	34.1884	133.0750	443.7844

At $n = 20$ the plain string departed 25.140729 times as much as the wound string. Finite-difference eigenvalues of the wound string with 1500 points were 110.0028, 550.3524, 1102.8161 and 2222.4382 Hz at $n = 1, 5, 10$ and 20, against 110.0028, 550.3549, 1102.8363 and 2222.6038 Hz from the formula, with a largest relative difference of 7.45×10^{-5} .

Observation 3.3

with the same weight, confining bending stiffness to the core makes overtone departures far smaller. The stiffness coefficient scales as the fourth power of diameter, so with the core diameter 0.4/0.955522 of the plain string's, the coefficient became 0.030710 times as large: the departure of the 20th tone fell from 443.7844 cents to 17.6520 cents.

4. The Separator

Observation 4.1

the separator is whether weight and bending stiffness are carried by one cross-section or split between separate parts. In a plain string the same cross-section carries both weight and bending stiffness, so making it heavy for a low note also makes it stiffer (Paper 942): in the wound string, 0.824758 of the pitch-setting weight was given to the winding and the bending stiffness to a thin core. The tension stayed at 114.403 N while the stiffness coefficient became 0.030710 times as large.

Observation 4.2

what is not said. The winding's real contribution to bending stiffness, real guitar and piano strings, tuning and how the tone is heard are not treated. What is shown is that in a model setting the winding's contribution to zero, splitting bending stiffness into the core while keeping the weight reduces overtone departures by the fourth power of the ratio of core to plain-string diameter.

5. Roll Call and Closing

1. linear density and stiffness coefficient of a wound string (Section 1): coefficient ratio is the fourth power of the diameter ratio.
2. tension 114.403 N for both (Computation 2.1): same linear density, same pitch.
3. winding share 0.824758 (Computation 2.1): the winding carries the weight.
4. coefficient ratio 0.030710 (Computation 3.1): only the core bends stiffly.
5. 17.6520 against 443.7844 cents at $n = 20$ (Computation 3.2): matches finite differences.
6. one cross-section, or separate parts (Observation 4.1): the separator; the scope notes of Papers 942 and 857.

Closing: A wound string carries its weight in the winding, and its overtones stray far less than a plain string of the same weight. A string with 0.3 mm copper wound on a 0.4 mm core had the same linear density as a plain steel string 0.955522 mm in diameter, and at the same 110 Hz the same tension of 114.403 N: its stiffness coefficient was 0.030710 times as large, and the departure of the 20th tone fell from 443.7844 cents to 17.6520 cents. The separator is whether weight and bending stiffness are carried by one cross-section or split between separate parts. Placed among the earlier papers: the overtone departures that Paper 942 counted for plain strings were counted for the wound strings that Papers 942 and 857 left out, showing they can be made small at the same weight. To be honest: the

winding does have some bending stiffness. In a model setting it to zero, how much splitting weight and stiffness reduces the departures, and no further.

References

1. Fletcher, H. (1964). Normal vibration frequencies of a stiff piano string. *The Journal of the Acoustical Society of America*.
 2. Fletcher, N. H., & Rossing, T. D. (1998). *The Physics of Musical Instruments* (2nd ed.). Springer.
 3. Yamagishi, Y. (2026). *Pitch Sees Only the Ratio of Tension to Weight, but the Instrument Bears the Tension Itself* [Paper 857]. YOI: <https://www.yamagishiyuuki.com/yoi/857>
 4. Yamagishi, Y. (2026). *Bending Stiffness Sharpens Overtones Past Whole Multiples, and at the Same Pitch a Thicker String Strays Further* [Paper 942]. YOI: <https://www.yamagishiyuuki.com/yoi/942>
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