

A Reddened Star's Color Temperature Reads Lower the Hotter the Star, and Never Exceeds a Ceiling However Hot

— extinction shrinks the brightness ratio of two colors by the same factor at every temperature, but for hot stars that becomes a large drop in temperature —the separator is whether the shift is counted on the ratio or on the temperature — [Paper 959]

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— *From thinking to syncing.* —

Abstract

When a star's surface temperature is read from its color, dust between the star and us dims blue light more strongly and reddens the color. This paper multiplies the black-body brightness ratio at 440 nm and 550 nm of Paper 943 by an extinction inversely proportional to wavelength (a set form), and solves, without relying on a first-order estimate, how far the temperature read from the apparent ratio differs from the true temperature. No new theorem or law is claimed. First, extinction shrinks the ratio by a factor independent of temperature: for extinction of 0.1, 0.3 and 1.0 magnitudes at 550 nm, the difference in extinction between the two wavelengths (the color excess) is 0.250000 times that, and the factor on the ratio was 0.977237, 0.933254 and **0.794328** (Section 2). Second, the hotter the star, the further its temperature reads low: with 1.0 magnitude of extinction, the apparent temperature was 0.876233 of the true temperature at 4000 K, 0.828550 at 5772 K and **0.339184** at 40000 K (13567.372 K) (Section 2). Third, the first-order estimate from the sensitivity at that temperature misses badly for hot stars: using the sensitivity of Paper 943, the first-order estimate gives 0.103318 at 1.0 magnitude and 40000 K, below the actual 0.339184. In all fifteen cases the first-order estimate came out lower (Section 3). Fourth, the apparent temperature has a ceiling: however high the true temperature, the apparent ratio does not exceed the infinitely hot limit ratio 2.441406 times the extinction factor. The ceiling on the apparent temperature was 146755.189, 51795.827 and **17921.911** K for 0.1, 0.3 and 1.0 magnitudes, and at 1.0 magnitude even a true 1000000 K reached only 0.988172 of the ceiling (Section 4). Fifth, the separator is whether the shift is counted on the ratio or on the temperature: on the ratio, the shift at 1.0 magnitude is the same 0.794328 at every temperature. On the temperature, the same shift spreads from 0.876233 to 0.339184, and even produces a ceiling (Section 5). **Keywords:** interstellar extinction, reddening, color excess, color temperature, black-body radiation, Planck's law.

Scope of this paper (scope note): No new theorem or law is claimed: that interstellar dust dims starlight and dims the blue side more strongly, reddening the color (interstellar extinction and reddening), that the difference in extinction at two wavelengths is called the color excess, and that a temperature read from color comes out low under extinction, are all known. The form of extinction is set: the extinction at each wavelength is set to the extinction at 550 nm times the ratio of 550 nm to the wavelength. The shape of real extinction curves, the values that fix that shape, and filter widths are not treated. Only an ideal black body at two wavelengths: as in Paper 943, the ratio of brightness at the two points 440 nm and 550 nm, without entering color index values. How the amount of extinction is found is not treated: distances to stars, estimates of the amount of dust, procedures for correcting extinction, and the temperatures of individual stars are not treated. Relation to earlier papers: Paper 943, in its scope note, did not treat observation, leaving out extinction by the atmosphere and interstellar dust, departures of starlight from a black body, and scatter in measuring instruments; this paper sits in the seat of extinction by interstellar dust (no earlier occupant). Paper 943 showed that the brightness ratio of two colors approaches a fixed value for hotter stars, that the ratio's sensitivity to temperature falls, and that a 1 percent error in the ratio becomes, to first order, an error in temperature of the reciprocal of the sensitivity in percent. Here the drop in temperature when the ratio shrinks by a large amount, as under extinction, is solved without relying on first order, and a ceiling on the apparent temperature is counted. What is added: showing that extinction shrinks the ratio by a factor independent of temperature, solving the apparent temperature at five temperatures and three extinctions, setting out the differences from the first-order estimate, deriving the ceiling on the apparent temperature, and placing the separator on "counted on the ratio, or on the temperature".

1. Notation and the Question

Setting: as in Paper 943, the ratio of black-body brightness (per wavelength) at 440 nm and 550 nm is

$$R(T) = \left(\frac{550}{440} \right)^5 \frac{e^{c_2/(\lambda_{550}T)} - 1}{e^{c_2/(\lambda_{440}T)} - 1}, \quad c_2 = 1.43877688 \times 10^{-2} \text{ m K}$$

(reproducing 0.975636 at 5772 K and 2.227526 at 40000 K). The extinction (in magnitudes) at wavelength λ is set to $A_\lambda = A_{550}(550\text{nm}/\lambda)$. The difference in extinction between the two wavelengths (the color excess), and the factor on the ratio, are

$$E = A_{440} - A_{550} = A_{550} \left(\frac{550}{440} - 1 \right), \quad R_{\text{app}} = R(T) \times 10^{-0.4E}$$

The apparent temperature T' solves $R(T') = R_{\text{app}}$ (the ratio rises with temperature, so bisection is used; the ratio at 12345 K returned 12345.000000 K). With the sensitivity $S = d \ln R / d \ln T$ of Paper 943, the first-order estimate is

$$\frac{T'}{T} \approx \exp \left(\frac{-0.4E \ln 10}{S(T)} \right)$$

Question: for the same extinction, how far does the apparent temperature differ depending on the true temperature, and is there a ceiling?

2. Shrinking the Ratio and the Apparent Temperature

Computation 2.1: the color excess is 0.250000 times A_{550} ; for $A_{550} = 0.1, 0.3$ and 1.0 magnitudes, $E = 0.025, 0.075$ and 0.250 magnitudes, and the factor on the ratio is 0.977237, 0.933254 and 0.794328, independent of temperature.

Computation 2.2: apparent temperature (K) and its fraction of the true temperature.

true temperature	$A_{550} = 0.1$ mag	0.3 mag	1.0 mag
4000 K	3944.218 (0.986055)	3837.248 (0.959312)	3504.933 (0.876233)
5772 K	5654.138 (0.979580)	5432.906 (0.941252)	4782.391 (0.828550)
10000 K	9617.953 (0.961795)	8943.650 (0.894365)	7225.123 (0.722512)
20000 K	18257.844 (0.912892)	15628.422 (0.781421)	10662.154 (0.533108)
40000 K	32722.404 (0.818060)	24341.623 (0.608541)	13567.372 (0.339184)

Observation 2.3

for the same extinction, the hotter the star, the smaller the fraction. For all three extinctions the fraction fell as the true temperature rose from 4000 K to 40000 K: at 1.0 magnitude a 4000 K star read as 0.876233 of its temperature and a 40000 K star as 0.339184. The factor on the ratio is the same 0.794328 at all five temperatures.

3. Against the First-Order Estimate

Computation 3.1: the sensitivity S of Paper 943 is 1.627815, 1.103448, 0.576885, 0.238265 and 0.101438 at 4000, 5772, 10000, 20000 and 40000 K (at 20000 K the formula 0.238265412 agrees with the difference quotient 0.238265631 within 10^{-6}). First-order fraction and solved fraction:

true temperature	$A_{550} = 0.1$ mag	0.3 mag	1.0 mag
4000 K	0.985954 / 0.986055	0.958452 / 0.959312	0.868096 / 0.876233
5772 K	0.979349 / 0.979580	0.939318 / 0.941252	0.811662 / 0.828550
10000 K	0.960872 / 0.961795	0.887149 / 0.894365	0.670896 / 0.722512
20000 K	0.907883 / 0.912892	0.748325 / 0.781421	0.380452 / 0.533108
40000 K	0.796926 / 0.818060	0.506120 / 0.608541	0.103318 / 0.339184

Observation 3.2

the first-order estimate misses badly for hot stars and large extinction. In all fifteen cases the first-order estimate was lower than the solved fraction: at 4000 K and 0.1 magnitude it was almost the same, 0.985954 against 0.986055, but at 40000 K and 1.0 magnitude it was 0.103318 against 0.339184.

4. The Ceiling on the Apparent Temperature

Computation 4.1: the infinitely hot limit of the ratio is $(550/440)^4 = 2.441406$, and the ratio at 10^9 K was 2.441398. The ratio rises with temperature toward this value, so the apparent ratio does not exceed 2.441406 times the extinction factor. The ceiling on the apparent temperature solved from that value, and the apparent temperature for true 40000, 100000 and 1000000 K as a fraction of the ceiling:

extinction	ceiling	40000 K	100000 K	1000000 K
0.1 mag	146755.189 K	0.222973	0.419524	0.879001
0.3 mag	51795.827 K	0.469953	0.693179	0.958139
1.0 mag	17921.911 K	0.757027	0.890403	0.988172

Observation 4.2

the color temperature of a reddened star has a ceiling. For all three extinctions the apparent temperature approached the ceiling as the true temperature rose, and the ceiling was lower for larger extinction: at 1.0 magnitude no star, however hot, reads hotter than 17921.911 K, and a true 40000 K star is already at 0.757027 of the ceiling.

5. The Separator

Observation 5.1

the separator is whether the shift is counted on the ratio or on the temperature. On the ratio, the shift from extinction is the same factor at every temperature: 0.794328 at all five temperatures for 1.0 magnitude: on the temperature, the same shift spreads from 0.876233 at 4000 K to 0.339184 at 40000 K, and the limit of the ratio becomes a ceiling of 17921.911 K on the temperature.

Observation 5.2

what is not said. Real extinction curves, filters and color indices, how the amount of extinction is found and corrected, and the temperatures of individual stars are not treated. What is shown is that with a set form of extinction and an ideal black body at two wavelengths, the apparent temperature reads lower the hotter the star and does not exceed a ceiling.

6. Roll Call and Closing

1. color excess and the factor on the ratio (Section 1): independent of temperature.
2. factor on the ratio **0.794328** (Computation 2.1): extinction of 1.0 magnitude.
3. **0.339184** at 40000 K (Computation 2.2): hotter stars read lower.
4. first-order estimate **0.103318** (Computation 3.1): misses for hot stars.
5. ceiling **17921.911 K** (Computation 4.1): not exceeded however hot.
6. on the ratio, or on the temperature (Observation 5.1): the separator; the scope note of Paper 943.

Closing: A reddened star's color temperature reads lower the hotter the star, and never exceeds a ceiling however hot. Extinction of 1.0 magnitude at 550 nm shrank the brightness ratio at 440 nm and 550 nm to 0.794328 at every temperature: the apparent temperature became 0.876233 of the true value for a 4000 K star and 0.339184 for a 40000 K star, far from the first-order estimate of 0.103318. The ceiling on the apparent temperature was 146755.189, 51795.827 and 17921.911 K for 0.1, 0.3 and 1.0 magnitudes. The separator is whether the shift is counted on the ratio or on the temperature. Placed among the earlier papers: the ratio and sensitivity that Paper 943 counted for an ideal black body were multiplied by the interstellar extinction that Paper 943 set aside, and solved without relying on first order. To be honest: real extinction curves are not necessarily inversely proportional to wavelength. With a set form of extinction, how far the color temperature reads low and where it levels off, and no further.

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