

Cam Curves with the Same Peak Velocity Need Different Pitch Circles, Set by Where the Pressure Angle Peaks

— the pressure angle is the follower's velocity divided by the radius at that moment, and the cycloid reaches its largest angle before the lift is half done —the separator is whether the follower's velocity is read alone or divided by the cam's radius at that moment — [Paper 961]

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Abstract

When the angle between the direction in which a cam pushes its follower and the direction in which the follower moves (the pressure angle) is large, more of the push goes sideways on the follower, and in design the cam is made larger to hold the angle down. For the four motion curves of Paper 929, with a radially moving roller follower rising 10 mm over 60 degrees of cam angle, this paper finds the pitch circle radius needed to hold the largest pressure angle to 30 degrees (a set value). No new theorem or law is claimed. First, the needed pitch circle differs by 6.9276 mm between curves: the needed radius was 21.4575 mm for simple harmonic, 26.3648 mm for the 3-4-5 polynomial, 28.0797 mm for the parabola and **28.3851** mm for the cycloid (Section 2). Second, the order is the order of peak velocity: the peak velocity factor was 1.570796 for simple harmonic, 1.875000 for the 3-4-5 polynomial and 2.000000 for both parabola and cycloid, and the needed radius also rose in the order simple harmonic, 3-4-5 polynomial, parabola, cycloid (Section 2). Third, two curves with the same peak velocity need different radii: parabola and cycloid both have a peak velocity factor of 2.000000, but their needed radii differed by **0.3054** mm. The parabola's angle peaks at half the rise ($u = 0.5$), matching the closed form 28.0797 mm. The cycloid's angle peaked at $u = 0.4694$, with lift 0.4391 and velocity factor 1.981634 (Section 3). Fourth, the angle formula was checked by another construction: the largest pressure angle at a radius of 30 mm was 28.6202, 23.4132, 28.8218 and 27.3039 degrees from the angle formula, and 28.6201, 23.4132, 28.8218 and 27.3039 degrees from the slope of the pitch curve drawn in Cartesian coordinates, agreeing within 10^{-3} degrees (Section 2). Fifth, the separator is whether the follower's velocity is read alone or divided by the cam's radius at that moment: read alone, parabola and cycloid share the same 2.000000. Divided by the radius at that moment, the cycloid's angle peaks earlier, while the radius is still small, and the needed radius became 0.3054 mm larger (Section 4). **Keywords:** cam, pressure angle, pitch circle, follower, motion curve, cycloid.

Scope of this paper (scope note): No new theorem or law is claimed: the definition of a cam's pressure angle, enlarging the pitch circle (the base circle plus the roller radius) to hold the pressure angle down, rules of thumb for the allowed pressure angle, and the peak velocity of each motion curve are all known in cam design. These are set values: the 10 mm rise, 60 degrees of cam angle and 30 degrees of allowed pressure angle are set here. They correspond to no particular machine. Only a radial roller follower is treated: the follower's line of motion passes through the cam center; offset followers, flat-faced followers and swinging-arm followers are not included. Profile strength is not treated: contact stress, materials, undercutting where the roller radius exceeds the profile curvature, and the return segment are not treated. Relation to earlier papers: Paper 929, in its scope note, did not design cams, leaving out cam profile, pressure angle, contact stress and materials; this paper sits in the seat of pressure angle (no earlier occupant). Paper 929 set out the peak velocities of the four motion curves as 2, 1.570796, 2 and 1.875, and showed that the cycloid, despite its large peak acceleration, leaves less residual vibration because its acceleration does not jump at the ends. Here, for the same curves, the pressure angle that sets the size of the cam is counted. Paper 557 showed that a gear's pressure angle works in opposite directions for strengthening the tooth and for smoothing the drive: the pressure angle here is not that of a gear tooth but the angle at which a cam pushes its follower. What is added: finding the needed pitch circle radius for four curves, setting out where the angle peaks and the lift there, checking the parabola against a closed form, checking the angle formula against Cartesian slopes, and placing the separator on "velocity alone, or divided by the radius".

1. Notation and the Question

Setting: a rise of $h = 10$ mm takes place over a cam angle of $\beta = 60$ degrees ($\pi/3$ radians). The lift at dimensionless angle $u \in [0, 1]$ is $s(u)$, as in Paper 929, and the velocity factor is $v(u) = ds/du$. The follower is a radially moving roller; the pitch curve traced by the roller center has radius $r(u) = R_p + hs(u)$, where R_p is the pitch circle radius. The pressure angle ϕ is

$$\tan \phi(u) = \frac{1}{r} \frac{dr}{d\theta} = \frac{(h/\beta) v(u)}{R_p + h s(u)}$$

Its maximum was taken over u divided into 20000 steps, and R_p giving a maximum of 30 degrees was found by bisection. As another construction, the pitch curve was drawn in Cartesian coordinates and the largest pressure angle was also found from the angle between the curve's tangent and the circumferential direction.

Question: how much does the pitch circle that holds the largest pressure angle to 30 degrees differ between curves, and what sets it?

2. Needed Pitch Circles for Four Curves

Computation 2.1: needed pitch circle radius and where the angle peaks.

curve	peak velocity factor	needed radius (mm)	u at peak angle	lift there	velocity factor there
simple harmonic	1.570796	21.4575	0.4395	0.4055	1.542509
3-4-5 polynomial	1.875000	26.3648	0.4625	0.4299	1.853910
parabola	2.000000	28.0797	0.5000	0.5000	2.000000
cycloid	2.000000	28.3851	0.4694	0.4391	1.981634

Computation 2.2: largest pressure angle (degrees) at a pitch circle radius of 30 mm.

curve	angle formula	Cartesian slope
parabola	28.6202	28.6201
simple harmonic	23.4132	23.4132
cycloid	28.8218	28.8218
3-4-5 polynomial	27.3039	27.3039

All four agreed within 10^{-3} degrees. The peak velocity factors agreed with Paper 929's 2, 1.570796, 2 and 1.875 within 10^{-6} .

Observation 2.3

the order of needed pitch circles is the order of peak velocity. Both by peak velocity factor and by needed radius, the order from smallest was simple harmonic, 3-4-5 polynomial, parabola, cycloid: between the smallest, simple harmonic at 21.4575 mm, and the largest, cycloid at 28.3851 mm, a difference of 6.9276 mm, came the 3-4-5 polynomial and the parabola.

3. Two Curves with the Same Peak Velocity

Computation 3.1: for $u \leq 0.5$ the parabola has $v = 4u$ and $s = 2u^2$; its angle peaks at $u = 0.5$, and the needed radius is

$$R_p = \frac{2(h/\beta)}{\tan 30^\circ} - \frac{h}{2} = 28.0797 \text{ mm}$$

agreeing with the searched 28.0797 mm within 10^{-3} mm. The cycloid's needed radius was 0.3054 mm larger than the parabola's, and its angle peaked at $u = 0.4694$, before 0.5.

Observation 3.2

with the same peak velocity, a different position of the peak angle means a different needed radius. The parabola's angle peaks at $u = 0.5$, where its velocity also peaks, with lift 0.5000: the cycloid's angle peaked at $u = 0.4694$, where the lift was 0.4391, not yet half, and the velocity factor was 1.981634. For simple harmonic and the 3-4-5 polynomial too, the angle peaked at $u = 0.4395$ and 0.4625, before the velocity peak at $u = 0.5$.

4. The Separator

Observation 4.1

the separator is whether the follower's velocity is read alone or divided by the cam's radius at that moment. Read alone, parabola and cycloid share the same peak of 2.000000 and cannot be told apart: the pressure angle is the velocity factor divided by $R_p + hs$, so at the same velocity the angle is larger where the lift is smaller. The cycloid's angle peaked at a lift of 0.4391, and its needed radius of 28.3851 mm exceeded the parabola's 28.0797 mm.

Observation 4.2

what is not said. Contact stress, materials, undercutting, the return segment, offset or flat-faced followers, and what the allowed pressure angle should be are not treated. What is shown is how far, for a set rise and cam angle, the pitch circle holding the largest pressure angle to 30 degrees differs between curves, and that even two curves with the same peak velocity differ by where the angle peaks.

5. Roll Call and Closing

1. the pressure angle formula (Section 1): velocity factor divided by the radius.
2. needed radius from 21.4575 to **28.3851** mm (Computation 2.1): same order as peak velocity.
3. two constructions within 10^{-3} degrees (Computation 2.2): check of the angle formula.

4. parabola and cycloid differ by **0.3054 mm** (Computation 3.1): **same peak velocity 2.**
5. cycloid's angle peaks at $u = 0.4694$, lift **0.4391** (Observation 3.2): **before half.**
6. velocity alone, or divided by the radius (Observation 4.1): **the separator; the scope note of Paper 929.**

Closing: Cam curves with the same peak velocity need different pitch circles, set by where the pressure angle peaks. For a 10 mm rise over 60 degrees of cam angle, the pitch circle radius holding the largest pressure angle to 30 degrees was 21.4575 mm for simple harmonic, 26.3648 mm for the 3-4-5 polynomial, 28.0797 mm for the parabola and 28.3851 mm for the cycloid: parabola and cycloid both have a peak velocity factor of 2.000000, but the cycloid's angle peaked at a lift of 0.4391 and its needed radius was 0.3054 mm larger. The separator is whether the follower's velocity is read alone or divided by the cam's radius at that moment. Placed among the earlier papers: the four curves that Paper 929 compared by vibration were compared by pressure angle and cam size, which Paper 929 did not treat. To be honest: the allowed pressure angle and the rise are set values, and contact strength was not examined. For four curves, how much the cam size holding the angle differs, and no further.

References

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