

The Air and Plate Resonances of a Body Never Come Closer Than the Plate's Frequency on the Air Spring Alone

— the gap between the two resonances is the hypotenuse whose sides are the plate's own spring frequency minus the Helmholtz frequency and the plate's frequency on the air spring alone —the separator is whether the plate's frequency is counted with the cavity's air spring included or left out — [Paper 962]

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— *From thinking to syncing.* —

Abstract

A cavity with an opening sounds at one pitch (Paper 888), but in a real body the wall plates move too and couple to the air in the neck through the air in the cavity. This paper takes the cavity of Paper 888 (volume 0.010 m^3 , opening radius 0.05 m), lets part of its wall be a plate of area 0.04 m^2 and mass 0.2 kg (set values) moving like a piston, varies the plate's spring, and counts how close the two coupled resonances can come. No new theorem or law is claimed. First, the two resonances are pushed outside the original two frequencies: holding the plate still returns Paper 888's 163.086441 Hz . With the plate moving, for all seven plates the lower resonance was below the smaller of the Helmholtz frequency and the frequency of the plate with the opening blocked, and the upper resonance above the larger (Section 2). Second, the gap between the two frequencies is the hypotenuse of a right triangle: with f_v the plate's frequency on its own spring (no air), the gap is the hypotenuse whose sides are f_v minus the Helmholtz frequency and the plate's frequency on the air spring alone, 53.487201 Hz , agreeing within 10^{-6} Hz for seven plates (Section 2). Third, at their closest, the gap is the plate's frequency on the air spring alone: searching the plate spring without the formula, the smallest gap was 53.487201 Hz , with f_v at 163.086443 Hz , level with the Helmholtz frequency. With plate masses of 0.1 and 0.4 kg the smallest gap was 75.642324 and 37.821162 Hz (Section 3). Fourth, matching the blocked-opening plate frequency does not bring them closest: matching the plate frequency with the opening blocked, air spring included, to the Helmholtz frequency gave resonances of 133.694241 and 187.936757 Hz , a gap of 54.242515 Hz , 1.014121 times the smallest. At the smallest gap the blocked-opening plate frequency was 171.633530 Hz (Section 3). Fifth, the separator is whether the plate's frequency is counted with the cavity's air spring included or left out: matching the frequency counted without it gives the smallest gap; matching the frequency counted with it gives more than the smallest. The difference between the two ways of counting is exactly the plate's frequency on the air spring alone, and that became the smallest gap (Section 4). **Keywords:** Helmholtz resonance, coupled oscillators, resonating body, air spring, natural frequency, instrument body.

Scope of this paper (scope note): No new theorem or law is claimed: representing the low range of an instrument body by a model of two coupled oscillators, the cavity air and a plate, that coupled resonances are pushed outside the original frequencies, and that the product and sum of squares of the two frequencies are fixed by the original values, are all known in vibration and musical acoustics. The plate moves in one shape: the plate moves uniformly like a piston; its bending shape, multiple vibration shapes, losses between plate and air, and sound radiation are not included. These are set values: the cavity is as in Paper 888 (speed of sound 343.0 m/s, air density 1.2 kg/m³, volume 0.010 m³, opening radius 0.05 m, plate thickness 0.003 m, end correction 1.7 times the radius), and the plate area 0.04 m² and mass 0.2 kg are set here. They correspond to no instrument body. Timbre is not treated: how strongly each resonance sounds, how it is heard, and values for real guitars or drums are not treated. Relation to earlier papers: Paper 888, in its scope note, did not include plate vibration, leaving out coupling of the body's plates to the cavity and absorption at the walls; this paper sits in the seat of plates vibrating and coupling to the cavity (no earlier occupant). Paper 888 showed that the pitch of a cavity with an opening is set not by length, as in a pipe, but by volume and opening, whatever the shape. Here it is counted how, when the cavity's wall moves, the pitch splits in two and how close the two can come. What is added: finding the two resonances with a coupled plate for seven plates, confirming that the gap between them is a hypotenuse, searching for the smallest gap without the formula, comparing with matching the blocked-opening plate frequency, and placing the separator on "air spring included, or left out".

1. Notation and the Question

Setting: for the cavity of Paper 888 (speed of sound $c = 343.0$ m/s, air density $\rho = 1.2$ kg/m³, volume $V = 0.010$ m³, opening radius 0.05 m, plate thickness 0.003 m, with 1.7 times the radius added to the neck length), the neck air mass is $m_n = 8.293805 \times 10^{-4}$ kg and the cavity spring coefficient is $\kappa = \rho c^2 / V = 1.411788 \times 10^7$ Pa/m³. With outward displacement x of the neck air (opening area S_n) and outward displacement y of the wall plate (area $S_p = 0.04$ m², mass $m_p = 0.2$ kg, plate spring k_p), the cavity pressure is $p = -\kappa(S_n x + S_p y)$, and

$$m_n \ddot{x} = S_n p, \quad m_p \ddot{y} = -k_p y + S_p p$$

In angular frequency, with $\omega_H^2 = S_n^2 \kappa / m_n$ (Helmholtz), $\omega_v^2 = k_p / m_p$ (plate spring alone, no air), $\omega_a^2 = S_p^2 \kappa / m_p$ (plate on the air spring alone) and $\omega_b^2 = \omega_v^2 + \omega_a^2$ (plate with the opening blocked), the two coupled angular frequencies satisfy

$$\omega_1^2 + \omega_2^2 = \omega_H^2 + \omega_b^2, \quad \omega_1 \omega_2 = \omega_H \omega_v, \quad (\omega_2 - \omega_1)^2 = (\omega_H - \omega_v)^2 + \omega_a^2$$

In the check, eigenvalues were found numerically and compared with these.

Question: however the plate spring is chosen, how close can the two resonances come?

2. The Two Resonances and Their Gap

Computation 2.1: Helmholtz $f_H = 163.086441$ Hz; plate on the air spring alone $f_a = 53.487201$ Hz. The two resonances for seven choices of the plate's own spring frequency f_v :

f_v (Hz)	f_b (Hz)	lower	upper	gap	hypotenuse
80.000000	96.233469	75.043614	173.857769	98.814156	98.814156
120.000000	131.380671	109.706192	178.388956	68.682764	68.682764
140.000000	149.869545	124.756476	183.013359	58.256883	58.256883
163.086441	171.633528	138.521059	192.008259	53.487201	53.487201
180.000000	187.778808	145.566533	201.664207	56.097675	56.097675
220.000000	226.408658	154.349847	232.452560	78.102713	78.102713
300.000000	304.730833	159.587324	306.577811	146.990487	146.990487

The hypotenuse is $\sqrt{(f_H - f_v)^2 + f_a^2}$. For seven plates the gap agreed with it within 10^{-6} Hz, the product of the two resonances agreed with $f_H f_v$ within a relative 10^{-9} (13046.915309 and 13046.915309 at $f_v = 80$ Hz), and the lower resonance was below the smaller of f_H and f_b and the upper above the larger. With a plate spring of 10^{12} N/m the lower resonance was 163.086439 Hz.

Observation 2.2

the two resonances are pushed apart, and their gap is a hypotenuse. The gap was smallest in the row with f_v level with Helmholtz, and larger in rows further away: the gap was 146.990487 Hz at $f_v = 300$ Hz and 98.814156 Hz at $f_v = 80$ Hz. In the row with f_v level with Helmholtz, the gap was the same 53.487201 Hz as f_a .

3. The Plate That Comes Closest

Computation 3.1: without the formula, f_v was searched between 50 and 400 Hz by golden-section search (200 steps) to minimize the gap between numerical eigenvalues.

matched frequency	two resonances (Hz)	gap (Hz)	ratio to smallest
smallest gap ($f_v = 163.086443$ Hz, $f_b = 171.633530$ Hz)	138.521060, 192.008260	53.487201	1
blocked-opening plate $f_b = f_H$	133.694241, 187.936757	54.242515	1.014121

The smallest gap found agreed with f_a within 10^{-6} Hz, and f_v there with f_H within 10^{-3} Hz. With plate masses of 0.1 and 0.4 kg and $f_v = f_H$, the gap was 75.642324 and 37.821162 Hz, within 10^{-6} Hz of each f_a (75.642324 and 37.821162 Hz).

Observation 3.2

the two resonances come no closer than the plate's frequency on the air spring alone. They come closest when the plate's frequency without air, f_v , is level with Helmholtz: matching the blocked-opening plate frequency f_b instead gave a gap of 54.242515 Hz, 1.014121 times the smallest. A lighter plate raises f_a , and the smallest gap widened to 75.642324 Hz at 0.1 kg.

4. The Separator

Observation 4.1

the separator is whether the plate's frequency is counted with the cavity's air spring included or left out. Matching f_v , counted without it, to Helmholtz gave the smallest gap of 53.487201 Hz: matching f_b , counted with it, gave 54.242515 Hz. The difference of the squares of the two ways of counting, $\omega_b^2 - \omega_v^2$, is ω_a^2 of the plate on the air spring alone, and its f_a became the smallest gap.

Observation 4.2

what is not said. Plate bending shapes, multiple vibration shapes, losses and radiation, how strongly each resonance sounds, and values for real instruments are not treated. What is shown is that in a model coupling a piston-like plate to the cavity of Paper 888, the gap between the two resonances is a hypotenuse and comes no closer than the plate's frequency on the air spring alone.

5. Roll Call and Closing

1. sum, product and gap of the two angular frequencies (Section 1): **the gap is a hypotenuse.**
2. gap and hypotenuse within 10^{-6} Hz for seven plates (Computation 2.1): **pushed apart.**
3. smallest gap **53.487201** Hz (Computation 3.1): **the plate on the air spring alone.**
4. matching the blocked-opening plate gives **1.014121** times (Computation 3.1): **not the smallest.**
5. **75.642324** and **37.821162** Hz at **0.1** and **0.4** kg (Computation 3.1): **the smallest gap is still f_a .**
6. air spring included, or left out (Observation 4.1): **the separator; the scope note of Paper 888.**

Closing: The air and plate resonances of a body never come closer than the plate's frequency on the air spring alone. Coupling a plate of area 0.04 m^2 and mass 0.2 kg to the 163.086441 Hz cavity of Paper 888, the gap between the two resonances became the hypotenuse whose sides are the plate's own spring frequency minus Helmholtz and the plate's frequency on the air spring alone, 53.487201 Hz : the smallest gap was 53.487201 Hz , reached when the plate's frequency without air was level with Helmholtz. Matching the blocked-opening plate frequency gave a gap of 54.242515 Hz . The separator is whether the plate's frequency is counted with the cavity's air spring included or left out. Placed among the earlier papers: to the pitch of the cavity that Paper 888 counted with the plates still, the plate vibration that Paper 888 did not include was coupled. To be honest: real body plates bend and vibrate in many shapes. With one piston-like plate, how close the two resonances can come, and no further.

References

1. Fletcher, N. H., & Rossing, T. D. (1998). *The Physics of Musical Instruments* (2nd ed.). Springer.
2. Kinsler, L. E., Frey, A. R., Coppers, A. B., & Sanders, J. V. (2000). *Fundamentals of Acoustics* (4th ed.). Wiley.
3. Yamagishi, Y. (2026). *A Resonating Body's Pitch Is Set Not by Length, as in a Pipe, but by Volume and Opening, Whatever the Shape* [Paper 888]. YOI: <https://www.yamagishiyuuki.com/yoi/888>

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