

Doubling the Pages, Which Pages Share a Side Is Still Set by Remainder Mod 4, and a Neighbor Sum Names the Fold That Joins Them

— across 28 folding orders of 32-page and 64-page signatures, the power of 2 dividing a neighbor sum minus 1 showed at which fold the pair was joined —the separator is whether pages are seen by their position on the sheet or by their remainder and a power of 2 — [Paper 964]

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Abstract

When one sheet is folded in half repeatedly into a section of a book (a signature), the order of pages on the sheet changes with the folding order, but at 16 pages which pages share a side and the sum 17 across the last fold did not change (Paper 877). This paper extends the folding model of Paper 877 to 32 and 64 pages, tries every sheet shape and folding order, and counts what decides which pages share a side and what neighbor sums decide. No new theorem or law is claimed. First, the folding orders differ by shape, and every order gives a different layout: for 4×2 (16 pages), 8×2 and 4×4 (32 pages), and 16×2 and 8×4 (64 pages) there were 3, 4, 6, 5 and 10 folding orders (28 in all), and no two layouts were the same (Section 2). Second, which pages share a side is set by remainder mod 4: for all five shapes and every folding order, the front side carried exactly the half of the pages with remainder 2 or 3 on division by 4, and the back side the pages with remainder 0 or 1 (Section 2). Third, a neighbor sum names the fold that joins the pair: taking the exponent of the power of 2 dividing a neighbor sum minus 1, and subtracting 1, gave the fold at which the two were joined. Across 2024 pairs checked there were 0 mismatches (Section 3). Fourth, the sum across the last fold is only the page count plus 1: at 16, 32 and 64 pages, the neighbor sums joined by the last fold were only 17, 33 and 65, and no other sum appeared (Section 3). Fifth, the separator is whether pages are seen by their position on the sheet or by their remainder and a power of 2: position on the sheet changes across the 28 orders. The remainder mod 4 of the page number decided the side, and the power of 2 in a neighbor sum minus 1 decided the joining fold, regardless of folding order or sheet shape (Section 4). **Keywords:** signature, imposition, bookbinding, page order, power of two, remainder.

Scope of this paper (scope note): No new theorem or law is claimed: folding one sheet into a signature, that the imposition layout changes with the folding method, and that pages back to back or across a fold follow rules for their sums, are all known in printing and binding. Folding follows the convention of Paper 877: the right (lower) half is turned over onto the left (upper) half, and pages are numbered from the top of the folded stack. Other ways of stacking and folds with cuts are not included. Binding practice is not treated: bleed, gripper margin, creep from paper thickness and the plate layout of real presses are not treated. The count stops at 64 pages. What was checked is five sheet shapes and 28 folding orders; larger signatures were not counted. Relation to earlier papers: Paper 877, in its scope note, did not extend the size, counting only the 16-page signature of a 4×2 grid folded three times and leaving out 32 pages or more, bleed and gripper margin; this paper sits in the seat of 32 pages or more (no earlier occupant). Paper 877 showed, for three folding orders of 16 pages, that the layout on the sheet changes but which pages share a side and the sum 17 across the last fold do not. Here it is counted whether the same holds at 32 and 64 pages, and the remainder that decides the side and the power of 2 that decides the joining fold are derived. What is added: trying every folding order for five sheet shapes, confirming that pages sharing a side are set by remainder mod 4, confirming over 2024 pairs that the power of 2 in a neighbor sum names the joining fold, confirming that the sum across the last fold is only the page count plus 1, and placing the separator on “position, or remainder and a power of 2”.

1. Notation and the Question

Setting: a sheet with a $W \times H$ grid on its front (side A) and back (side B) is folded in half, as in Paper 877, vertically or horizontally (the right or lower half turned over onto the upper). From the top of the folded stack, the upper face of each cell is numbered $1, 3, 5, \dots$ and the lower face $2, 4, 6, \dots$. The page count is $N = 2WH$ and the number of folds is $\log_2(WH)$.

For two cells adjacent on the sheet, let k be the fold at which they first come to the same position, and let s be the sum of their two pages on the same side. Write $\nu_2(s - 1)$ for the exponent of the power of 2 dividing $s - 1$.

Question: when the pages grow to 32 and 64, what do the pages sharing a side and the neighbor sums decide, independent of folding order and sheet shape?

2. Pages Sharing a Side

Computation 2.1: folding orders by sheet shape, and the front-side pages.

sheet	pages	folding orders	distinct layouts	front-side pages mod 4
4×2	16	3	3	2, 3
8×2	32	4	4	2, 3
4×4	32	6	6	2, 3
16×2	64	5	5	2, 3
8×4	64	10	10	2, 3

For every shape and order, 1 to N each appeared once, and pages back to back were an odd number and the next. The front of 4×2 was $\{2, 3, 6, 7, 10, 11, 14, 15\}$ as in Paper 877, and the front at 32 pages was $\{2, 3, 6, 7, 10, 11, 14, 15, 18, 19, 22, 23, 26, 27, 30, 31\}$, the same for all 10 orders of 8×2 and 4×4 . The back-side remainders were 0 and 1.

Observation 2.2

which pages share a side is set by remainder mod 4. For all five shapes and 28 orders, the front side was exactly the half of pages with remainder 2 or 3, where on the sheet a page is printed changes from order to order, but which side it is printed on can be read from its number alone.

3. Joining Folds and Neighbor Sums

Computation 3.1: for each folding order, the sums of neighbors joined at the k -th fold (excerpt).

sheet, order	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
4×2 V, H, V	5, 13	9, 25	17	(none)	(none)
8×2 V, H, V, V	37, 45	9, 25, 41, 57	17	33	(none)
4×4 H, V, H, V	21, 29, 37, 45	9, 25	17, 49	33	(none)
8×4 H, V, V, H, V	37 to 93	89, 105	17, 49	33, 97	65

(V is a vertical fold and H a horizontal fold.) Every sum at $k = 1$ was $4 \times \text{odd} + 1$ (5, 13, 21, 29, 37, 45), at $k = 2$ was $8 \times \text{odd} + 1$ (9, 25, 41, 57, 89, 105), at $k = 3$ was $16 \times \text{odd} + 1$ (17, 49), at $k = 4$ was $32 \times \text{odd} + 1$ (33, 97), and at $k = 5$

was 65. Across the five shapes and 28 orders, 2024 pairs of neighbor sums on the same side were checked, with 0 mismatches in $\nu_2(s - 1) - 1 = k$. The sums joined by the last fold were only 17, 33 and 65 at 16, 32 and 64 pages.

Observation 3.2

a neighbor sum names the fold that joins the pair. If the sum minus 1 is divisible by 4 but not by 8, the pair was joined at the first fold; if divisible by 8 but not by 16, at the second; at the last fold the sum minus 1 is divisible by the page count and smaller than twice the page count, so the sum can only be $N + 1$.

4. The Separator

Observation 4.1

the separator is whether pages are seen by their position on the sheet or by their remainder and a power of 2. Position on the sheet was different for all 28 orders of the five shapes: the remainder of the page number mod 4 decided the side, and the power of 2 in a neighbor sum minus 1 decided the joining fold, regardless of folding order or sheet shape.

Observation 4.2

what is not said. Bleed, gripper margin, paper thickness, other ways of stacking, signatures above 64 pages and the plate layout of real presses are not treated. What is shown is that, in a model folded by the convention of Paper 877, the side and the joining fold are set by page numbers for five shapes and 28 orders.

5. Roll Call and Closing

1. the folding model and $\nu_2(s - 1)$ (Section 1): the convention of Paper 877.
2. 28 folding orders, all layouts distinct (Computation 2.1): five sheet shapes.
3. front-side pages have remainder 2 or 3 mod 4 (Computation 2.1): independent of shape and order.
4. 0 mismatches in 2024 pairs (Computation 3.1): the neighbor sum names the joining fold.
5. last-fold sums 17, 33, 65 (Computation 3.1): only the page count plus 1.
6. position, or remainder and a power of 2 (Observation 4.1): the separator; the scope note of Paper 877.

Closing: Doubling the pages, which pages share a side is still set by remainder mod 4, and a neighbor sum names the fold that joins them. Folding five sheet shapes of 16, 32 and 64 pages in 28 orders by the convention of Paper 877, every layout on the sheet was different, yet the front side held the pages with remainder 2 or 3 mod 4: the exponent of the power of 2 dividing a neighbor sum minus 1, less 1, was the number of the joining fold, with 0 mismatches over 2024 pairs, and the last-fold sums were only 17, 33 and 65. The separator is whether pages are seen by their position on the sheet or by their remainder and a power of 2. Placed among the earlier papers. What Paper 877 found unchanged at 16 pages was counted at the 32 and 64 pages that Paper 877 did not treat. To be honest: neither paper thickness nor bleed is included. For one folding convention, how far page numbers decide the side and the fold, and no further.

References

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