For the Same Factor of Concentration Difference, a Cell Settles Faster When It Shrinks Than When It Swells

— the rate near the settled volume is proportional to the square of the outside concentration, so for a factor of 2 shrinking is 16 times faster —the separator is whether the force moving water is seen as a difference in volume or a difference in inside concentration — [Paper 966]

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September 2026
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— From thinking to syncing. —

Abstract

A cell moved into a concentrated solution shrinks, and into a dilute one it swells. The osmolality of normal saline was counted to be nearly equal to plasma (Paper 922), but how fast the cell volume changes was not treated.This paper takes a cell whose membrane passes only water (the osmotically inactive volume fraction b is a set value), moves it into a solution whose osmolality is r times 290 mOsm, and compares how fast it approaches its settled volume when swelling and when shrinking.No new theorem or law is claimed. First, the settled volume is set by the reciprocal of the factor: **with b = 0.4, the volume settled at 1.600000 times for 145 mOsm outside and 0.700000 times for 580 mOsm** (Section 2). **Second, for the same factor of 2, shrinking settles much sooner**: **with constant membrane area, swelling took 5.941899 times as long as shrinking to cover half the change, 9.255412 times to 90 percent, and 11.753247 times to 99 percent** (Section 2). **Third, the rate near the settled volume is proportional to the square of the outside concentration**: **the rate near the settled volume is r squared divided by $1 - b$, matching numerical slopes for b = 0.4 and 0.2, and the shrink-to-swell ratio was 16.000000 for a factor of 2, 5.062500 for 1.5 and 2.441406 for 1.25** (Section 3). **Fourth, shrinking is faster even when membrane area changes with volume**: **with area taken as volume to the 2/3 power, like a sphere, the time ratio for a factor of 2 was 5.050132 to half and 7.620672 to 99 percent, the near-settled rate ratio was 9.220873, and shrinking was faster in all six cases** (Section 3). **Fifth, the separator is whether the force moving water is seen as a difference in volume or a difference in inside concentration**: **a difference in volume looks the same for swelling and shrinking, but what moves water is the difference between inside and outside concentration, and the inside concentration is inversely proportional to the volume less b. The more concentrated the outside, the more a small volume change moves the inside concentration, and the sooner the force vanished** (Section 4).

Keywords: osmotic pressure, cell volume, water permeability, Boyle-van't Hoff relation, relaxation, membrane transport.

Scope of this paper (scope note): **No new theorem or law is claimed**: the formula taking water flow across a membrane as proportional to the osmotic difference, the relation giving the settled volume after subtracting osmotically inactive volume (the Boyle-van't Hoff relation), and that cells shrink in concentrated and swell in dilute solutions, are all known in physiology and membrane transport.**These are set values**: the osmotically inactive fraction b = 0.4 (and 0.2 as a check) and the original osmolality of 290 mOsm are set values, and time is in units of the reciprocal of water permeability times membrane area times temperature times 290 mOsm divided by volume. No value for any cell is claimed.**Only water crosses the membrane**: permeant solutes such as urea or glucose, the volume at which cells lyse, membrane tension and the cell's own volume regulation are not included.**Medical use is not discussed**, which solution to use when is not treated.**Relation to earlier papers**: **Paper 922, in its scope note, did not compute cell volume change, leaving out the speed of hemolysis, the speed of membrane permeation and the time for glucose to be taken up; this paper sits in the seat of the speed of cell volume change** (no earlier occupant).**Paper 922 showed that normal saline is six percent too concentrated by particle count but nearly equal to plasma once the osmotic coefficient is included**. **Here it is counted how fast the volume approaches a new balance when a cell is moved into a solution of different concentration.Paper 333 showed that the constant ratio of osmotic pressure to freezing-point depression follows from an assumption**. **Here too osmotic pressure is taken as proportional to concentration.**What is added: **finding settled volumes and times for each factor**, **confirming in six cases that shrinking settles sooner for the same factor**, **confirming by numerical slopes that the near-settled rate is proportional to the square of the outside concentration**, **confirming that changing the membrane area does not reverse the direction**, and **placing the separator on "difference in volume, or difference in inside concentration"**.

1. Notation and the Question

Setting: **write cell volume as its ratio x to the original volume, of which a fraction b is osmotically inactive. The inside solutes do not cross the membrane, so the inside osmolality is $290(1-b)/(x-b)$ mOsm. With the outside at $290r$ mOsm, water flow is proportional to the concentration difference:**

$$\frac{dx}{dt} = a(x)\left(\frac{1-b}{x-b} - r\right), \qquad x_\infty = b + \frac{1-b}{r}$$

Time is in units of the reciprocal of (water permeability)×(original membrane area)×RT×290 mOsm÷(original volume), and $a(x)$ is the membrane area ratio, set either constant ($a = 1$) or spherical ($a = x^{2/3}$). The times for the volume change $|x-1|$ to reach half, 90 percent and 99 percent of the full change are counted. Near the settled volume, the rate obtained by differentiating the right side in x and changing sign is

$$k = \frac{r^2}{1-b}\, a(x_\infty)$$

Question: **for the same factor of concentration difference, does a cell settle at the same rate when swelling and when shrinking?**

2. Swelling and Shrinking

Computation 2.1: **times for $b = 0.4$ with constant membrane area.**

outside (mOsm)	r	settled volume	to half	to 90%	to 99%
145.0	0.5000	1.600000	1.063600	4.446300	9.864500
580.0	2.0000	0.700000	0.179000	0.480400	0.839300
193.3	0.6667	1.300000	0.710800	2.703500	5.771500
435.0	1.5000	0.800000	0.251600	0.734100	1.360100
232.0	0.8000	1.150000	0.556100	1.990000	4.131800
362.5	1.2500	0.880000	0.314200	0.970600	1.863500

Computation 2.2: **swelling time divided by shrinking time for the same factor.**

membrane area, factor	to half	to 90%	to 99%	near-settled rate ratio (shrink/swell)
constant, 2	5.941899	9.255412	**11.753247**	16.000000
constant, 1.5	2.825119	3.682741	4.243438	5.062500
constant, 1.25	1.769892	2.050278	2.217226	2.441406
sphere, 2	5.050132	6.579169	7.620672	9.220873
sphere, 1.5	2.577335	3.020531	3.291023	3.662656
sphere, 1.25	1.683261	1.838691	1.927458	2.042506

> **Observation 2.3**
>
> **for the same factor, shrinking settles sooner.In all six cases, shrinking reached half, 90 percent and 99 percent sooner**: **the ratio grew closer to the settled volume: for constant area and a factor of 2, the time ratio to 99 percent was 11.753247, approaching the near-settled rate ratio of 16.000000.**

3. The Rate Near the Settled Volume

Computation 3.1: rates from slopes taken 10^{-6} above and below the settled volume, and the formula $r^2/(1-b)$.

b	r	numerical slope	formula
0.4	0.5	0.416667	0.416667
0.4	2.0	6.666667	6.666667
0.2	0.5	0.312500	0.312500
0.2	2.0	5.000000	5.000000

For constant area and a factor of 2, the shrink-to-swell rate ratio was 16.000000 for both $b = 0.4$ and 0.2. With spherical membrane area, the near-settled rate is the constant-area value times $x_\infty^{2/3}$, giving 0.569992 for 145 mOsm and 5.255823 for 580 mOsm.

> **Observation 3.2**
>
> the near-settled rate is proportional to the square of the outside concentration, and the ratio does not depend on b.With constant area the rate is $r^2/(1-b)$, so two solutions a factor q apart differ by q^4 (16 for 2, 5.062500 for 1.5, 2.441406 for 1.25): for a sphere, whose membrane area grows as it swells, the ratio shrank to 9.220873, but the direction did not change.

4. The Separator

> **Observation 4.1**
>
> the separator is whether the force moving water is seen as a difference in volume or a difference in inside concentration.Seen as a difference in volume, the departure from the settled volume seems to shrink in the same way whether swelling or shrinking. What moves water is the difference between inside and outside concentration, and the inside concentration is inversely proportional to $x - b$. In a concentrated outside solution $x_\infty - b$ is small, so a small volume change moves the inside concentration a lot, and the near-settled rate grows as r^2.

Observation 4.2

what is not said.Real values of b and water permeability, permeant solutes, the volume at lysis, membrane tension, volume regulation by the cell and medical use are not treated. What is shown is that in a membrane model passing only water, a cell settles sooner when shrinking than when swelling for the same factor, and the near-settled rate is proportional to the square of the outside concentration.

5. Roll Call and Closing

1. **water flow and settled volume** (Section 1): **inside concentration inversely proportional to $x - b$.**
2. **swelling takes 11.753247 times as long (to 99%) for a factor of 2** (Computation 2.2): **shrinking is sooner.**
3. **shrinking sooner in all six cases** (Computation 2.2): **constant and spherical area.**
4. **near-settled rate $r^2/(1-b)$, ratio 16.000000** (Computation 3.1): **independent of b.**
5. **ratio 9.220873 for a sphere** (Computation 3.1): **same direction.**
6. **difference in volume, or in inside concentration** (Observation 4.1): **the separator; the scope note of Paper 922.**

Closing: **For the same factor of concentration difference, a cell settles faster when it shrinks than when it swells.In a membrane model passing only water, with 145 and 580 mOsm outside, swelling took 5.941899 times (to half) to 11.753247 times (to 99 percent) as long as shrinking**: **the near-settled rate was $r^2/(1-b)$, the ratio for a factor of 2 was 16.000000 regardless of b, and even with spherical membrane area it was 9.220873 in the same direction.The separator is whether the force moving water is seen as a difference in volume or a difference in inside concentration.Placed among the earlier papers**: **when cells are moved into the solutions that Paper 922 compared by osmolality, the speed of volume change that Paper 922 did not compute was counted.To be honest**: **real cells also pass solutes and regulate their volume. With a membrane passing only water, how much shrinking and swelling differ, and no further.**

References

1. Atkins, P., & de Paula, J. (2006). *Physical Chemistry*. Oxford University Press.
2. Kedem, O., & Katchalsky, A. (1958). Thermodynamic analysis of the permeability of biological membranes to non-electrolytes. *Biochimica et Biophysica Acta*.
3. Yamagishi, Y. (2026). *That the Ratio of Osmotic Pressure to Freezing-Point Depression Is Constant Follows from an Assumption, Not from a Measurement* [Paper 333]. YOI: https://www.yamagishiyuuki.com/yoi/333; DOI: 10.5281/zenodo.22681740

4. Yamagishi, Y. (2026). *Normal Saline Is Six Percent Too Concentrated by Particle Count, but Nearly Equal to Plasma Once the Osmotic Coefficient Is Included* [Paper 922]. YOI: https://www.yamagishiyuuki.com/yoi/922

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