

The Roller Size That Avoids Cam Undercutting Is Ordered by Deceleration, Not by Pressure Angle

— the smallest convex radius of curvature is smaller for curves with larger peak deceleration, and simple harmonic motion, which needs the smallest cam for pressure angle, also allows the smallest roller on that cam —
the separator is whether the pitch curve is read by its slope or by its bending — [Paper 967]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

September 2026

YOI: <https://www.yamagishiyuuki.com/yoi/967>

— *From thinking to syncing.* —

Abstract

A cam profile lies inside the pitch curve traced by the roller center by one roller radius. If the roller is larger than the radius of curvature of a convex part of the pitch curve, the profile is cut away and the follower no longer moves as designed (undercutting). For the four motion curves of Paper 961 (a 10 mm rise over 60 degrees of cam angle), this paper finds the smallest convex radius of curvature of the pitch curve at the pitch circles Paper 961 found necessary for a 30 degree pressure angle and at a 30 mm pitch circle, and compares the order with that of pressure angle. No new theorem or law is claimed. First, at the same pitch circle, the smallest radius of curvature follows the order of peak deceleration: at a 30 mm pitch circle, the smallest radius of curvature was **16.4923** mm for the cycloid, **17.2563** mm for the 3-4-5 polynomial, **18.8235** mm for simple harmonic and **19.6172** mm for the parabola, in the same order as the peak deceleration factors **6.283185**, **5.773502**, **4.934801** and **4.000000** from largest (Section 2). Second, that order differs from the order for pressure angle: the pitch circle needed for a 30 degree pressure angle rose in the order simple harmonic, 3-4-5 polynomial, parabola, cycloid (Paper 961), but for the smallest radius of curvature the parabola was largest and simple harmonic third (Section 2). Third, simple harmonic, which needs the smallest cam for pressure angle, also allows the smallest roller: at the pitch circles of Paper 961, the smallest radius of curvature was **12.9428** mm for simple harmonic (21.4575 mm), **14.9088** mm for the 3-4-5 polynomial, **15.4845** mm for the cycloid and **18.3906** mm for the parabola (Section 3). Fourth, the smallest radius of curvature appears in the decelerating part near the end of the rise: at a 30 mm pitch circle, it appeared at $u = 0.7543$ (lift 0.9134) for the cycloid, $u = 0.7902$ for the 3-4-5 polynomial, at the end of the rise $u = 1$ for simple harmonic, and at $u = 0.5$, where deceleration begins, for the parabola. The formula and the smallest circle through three points of the curve drawn in Cartesian coordinates agreed within 0.01 mm (Section 2). Fifth, the separator is whether the pitch curve is read by its slope or by its bending: pressure angle is set by the slope of the pitch curve (the velocity factor), and undercutting by its bending (the acceleration factor). The two design limits ranked the same four curves in different orders (Section 4). **Keywords:** cam, undercutting, radius of curvature, roller follower, pitch curve, motion curve.

Scope of this paper (scope note): No new theorem or law is claimed: that undercutting occurs with a roller follower when the convex radius of curvature of the pitch curve is smaller than the roller radius, the formula for radius of curvature of a curve in polar coordinates, and the peak acceleration of each motion curve, are all known in cam design. The setting is that of Paper 961: the 10 mm rise, 60 degrees of cam angle, radially moving roller follower, four motion curves and needed pitch circles are those of Paper 961, and the 30 mm pitch circle is set for comparison. Profile strength is not treated: contact stress, materials, roller shaft strength and the return segment are not treated; curvature at the junctions with the dwells before and after the rise was seen only through the values at the ends of the rise. No design is recommended, which curve or roller to use is not stated. Relation to earlier papers: Paper 961, in its scope note, did not treat profile strength, leaving out contact stress, materials, undercutting where the roller radius exceeds the profile curvature, and the return segment; this paper sits in the seat of undercutting (no earlier occupant). Paper 961 showed that even among cam curves with the same peak velocity, the pitch circle holding the pressure angle changes with where the angle peaks. Here, for the same curves and pitch circles, the curvature limiting roller size is counted, and its order is shown to differ from that of pressure angle. Paper 929 set out the peak accelerations of the four motion curves. Here the decelerating side of those is used as what orders the smallest radius of curvature. What is added: finding the smallest convex radius of curvature for four curves and two pitch circles, checking it independently with circles through three points, showing that the order matches peak deceleration and differs from pressure angle, setting out where the minimum appears, and placing the separator on “slope, or bending”.

1. Notation and the Question

Setting: as in Paper 961, a rise of $h = 10$ mm takes place over a cam angle of $\beta = 60$ degrees, and the pitch curve has radius $r(\theta) = R_p + hs(u)$ with $u = \theta/\beta$. With velocity factor $v = ds/du$ and acceleration factor $a = d^2s/du^2$, $r' = dr/d\theta = (h/\beta)v$ and $r'' = (h/\beta^2)a$, and the radius of curvature of the polar curve is

$$\rho = \frac{(r^2 + r'^2)^{3/2}}{r^2 + 2r'^2 - r r''}$$

Over points where the denominator is positive (convex), the minimum of ρ was searched with u divided into 20000 steps; undercutting is taken to occur if the roller radius exceeds it. As a check, the pitch curve was drawn at 4001 points in Cartesian coordinates and the smallest radius of the circles through three neighboring points was also found.

Question: how does the roller size that avoids undercutting rank the four curves, and is it the same order as for pressure angle?

2. Curvature at the Same Pitch Circle

Computation 2.1: smallest convex radius of curvature at a 30 mm pitch circle.

curve	smallest radius (mm)	three-point circle (mm)	u at minimum	lift there	peak deceleration factor
cycloid	16.4923	16.4923	0.7543	0.9134	6.283185
3-4-5 polynomial	17.2563	17.2563	0.7902	0.9342	5.773502
simple harmonic	18.8235	18.8235	1.0000	1.0000	4.934801
parabola	19.6172	19.6181	0.5000	0.5000	4.000000

In all eight cases (four curves $\times$ two pitch circles), the formula and the three-point circles agreed within 0.01 mm.

Observation 2.2

at the same pitch circle, curves with stronger deceleration allow smaller rollers. The order of smallest radius of curvature from smallest (cycloid, 3-4-5 polynomial, simple harmonic, parabola) matched the order of peak deceleration factor from largest: compared with the order of pitch circle needed for a 30 degree pressure angle from smallest (simple harmonic, 3-4-5 polynomial, parabola, cycloid), the parabola and simple harmonic changed places.

3. Curvature at the Pitch Circles of Paper 961

Computation 3.1: smallest convex radius of curvature at the pitch circles needed for a 30 degree pressure angle (Paper 961).

curve	pitch circle (mm)	smallest radius (mm)	u at minimum	acceleration factor there
simple harmonic	21.4575	12.9428	1.0000	−4.934802
3-4-5 polynomial	26.3648	14.9088	0.7932	−5.771411
cycloid	28.3851	15.4845	0.7558	−6.279014
parabola	28.0797	18.3906	0.5000	−4.000000

Observation 3.2

simple harmonic, which needs the smallest cam for pressure angle, also allows the smallest roller on that cam. Simple harmonic held the pressure angle to 30 degrees with a 21.4575 mm pitch circle (Paper 961): on that small cam, deceleration at the end of the rise brought the radius of curvature down to 12.9428 mm, the smallest of the four. The parabola needs a 28.0797 mm cam, but its smallest radius of curvature was 18.3906 mm.

4. The Separator

Observation 4.1

the separator is whether the pitch curve is read by its slope or by its bending. Pressure angle is set by r'/r , that is, by the velocity factor, which favored simple harmonic with its small peak velocity (Paper 961): undercutting is set by the $-rr''$ in the denominator of the radius of curvature, that is, by the deceleration factor, which at the same pitch circle favored the parabola with its small peak deceleration. On one cam, the two limits ranked the curves in different orders.

Observation 4.2

what is not said. Contact stress, materials, roller shaft strength, the return segment, the detailed curvature at dwell junctions and which curve to use are not treated. What is shown is that, in the setting of Paper 961, the smallest convex radius of curvature of the pitch curve is ordered by deceleration and differs from the order for pressure angle.

5. Roll Call and Closing

1. radius of curvature in polar coordinates (Section 1): r'' , the acceleration factor, in the denominator.
2. cycloid smallest at **16.4923** mm on a 30 mm pitch circle (Computation 2.1): order of peak deceleration.
3. within 0.01 mm of three-point circles (Computation 2.1): check of the formula.
4. simple harmonic **12.9428** mm on the pitch circles of Paper 961 (Computation 3.1): smallest cam for pressure angle, smallest roller too.
5. order differs from pressure angle (Observation 2.2): parabola and simple harmonic swap.
6. slope, or bending (Observation 4.1): the separator; the scope note of Paper 961.

Closing: The roller size that avoids cam undercutting is ordered by deceleration, not by pressure angle. For the four motion curves of Paper 961, the smallest convex radius of curvature at a 30 mm pitch circle was 16.4923 mm for the cycloid, 17.2563 mm for the 3-4-5 polynomial, 18.8235 mm for simple harmonic and 19.6172 mm for the parabola, the order of peak deceleration factor: at the pitch circles Paper 961 needed for a 30 degree pressure angle, simple harmonic was smallest at 12.9428 mm, so the curve favored by pressure angle was disfavored by undercutting. The separator is whether the pitch curve is read by its slope or by its bending. Placed among the earlier papers: the four curves Paper 961 compared by pressure angle were compared by the undercutting Paper 961 did not treat, with the deceleration that sets curvature read from the acceleration ranking of Paper 929. To be honest: neither contact strength nor the return segment was examined. From the curvature of the rise, how usable roller size ranks, and no further.

References

1. Norton, R. L. (2009). *Cam Design and Manufacturing Handbook*. Industrial Press.
 2. Yamagishi, Y. (2026). *A Cycloidal Cam Has the Largest Peak Acceleration, Yet Leaves Less Vibration at Speed Because Its Acceleration Does Not Jump at the Ends* [Paper 929]. YOI: <https://www.yamagishiyuuki.com/yoi/929>
 3. Yamagishi, Y. (2026). *Cam Curves with the Same Peak Velocity Need Different Pitch Circles, Set by Where the Pressure Angle Peaks* [Paper 961]. YOI: <https://www.yamagishiyuuki.com/yoi/961>
-

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)