

The Smallest Pair of Tanks Puts the Smaller Tank First for a Second-Order Reaction and the Larger First for Order One Half

— for a first-order reaction equal sizes are smallest, and when the order departs from 1 an unequal pair is a few percent smaller —the separator is whether the reaction rate changes more steeply or more gently than the first power of concentration — [Paper 971]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

September 2026

YOI: <https://www.yamagishiyuuki.com/yoi/971>

— *From thinking to syncing.* —

Abstract

Splitting a well-mixed vessel into tanks in series reduces the total size needed to reach a given remainder, and the cost trade-off of the number of tanks has been counted (Papers 931 and 960). There the reaction was first order and all tanks had the same size. This paper takes two tanks and, for reaction orders 0.5, 1 and 2 and exit remainders of 0.1 and 0.01, finds the size split giving the smallest total residence time, and compares it with equal sizes. No new theorem or law is claimed. First, for a first-order reaction, equal sizes are smallest: equal tanks of 2.162278 each for remainder 0.1 and 9.000000 each for 0.01 were optimal, matching the values of Paper 960 (Section 2). Second, for a second-order reaction, the smaller tank comes first: at order 2 and remainder 0.1 the optimum was 11.029288 for the first tank and 15.917041 for the second (ratio **0.692923**), and at remainder 0.01, 280.917867 and 479.103817 (ratio **0.586340**) (Section 2). Third, for order one half, the larger tank comes first: at order 0.5 and remainder 0.1 the first tank was 1.070776 and the second 0.818074 (ratio **1.308899**), and at 0.01, 2.205858 and 1.388775 (ratio **1.588347**) (Section 2). Fourth, the gain is a few percent, larger for smaller remainders: the optimal total was 0.987031 (remainder 0.1) and **0.968676** (0.01) of the equal-size total at order 2, and 0.995511 and 0.979957 at order 0.5 (Section 3). Fifth, the separator is whether the reaction rate changes more steeply or more gently than the first power of concentration: at order 2 and remainder 0.1, the small first tank brought concentration from 1 down to 0.259170 and the large second tank down to 0.1. At order 0.5 the large first tank brought it down to 0.358698. At first order the intermediate concentration was 0.316228 and the two tanks were the same size (Section 4). **Keywords:** continuous stirred-tank reactor, tanks in series, reaction order, second-order reaction, residence time, reactor design.

Scope of this paper (scope note): No new theorem or law is claimed: the balance equation of a continuous stirred tank, that the optimal size split of tanks in series changes with reaction order, and that equal sizes are optimal for first order, are all known in reaction engineering. These are set values: an inlet concentration of 1, a rate constant of 1, orders 0.5, 1 and 2, and remainders of 0.1 and 0.01 are set values, corresponding to no reaction or equipment. Only two tanks are treated: three or more tanks, tanks at different temperatures, combination with plug flow and tank cost are not treated. No design is recommended, which split should be used is not stated. Relation to earlier papers: Paper 960, in its scope note, treated first-order reaction only, leaving out second-order reactions, tanks at different temperatures, and designs with tanks of unequal size; this paper sits in the seats of second-order reactions and unequal tank sizes (no earlier occupant). Paper 931, in its scope note, also treated first-order reaction only, leaving out second-order reactions; this paper sits in that seat as well. Paper 960 showed that the cheapest number of tanks in series scales with the decades to remove and that each tank's share is set by the cost exponent alone. Here the number of tanks is fixed at two and the size split is counted for each order. What is added: finding the optimal split for three orders and two remainders, confirming that equal sizes are optimal at first order, showing that which tank should be smaller reverses with order, setting out the gain over equal sizes, and placing the separator on "steeper or gentler than the first power".

1. Notation and the Question

Setting: inlet concentration 1, rate constant 1 and reaction rate c^n . The balance of a continuous stirred tank with inlet c_{in} , outlet c_{out} and residence time τ is

$$\tau = \frac{c_{in} - c_{out}}{c_{out}^n}$$

With intermediate concentration c_m between the two tanks, the total is $\tau_1 + \tau_2 = (1 - c_m)/c_m^n + (c_m - r)/r^n$; c_m was searched over 4001 points equally spaced in logarithm between r and 1 and refined by golden-section search. For the equal-size pair, the common τ giving outlet r was found by bisection.

Question: how does the size split minimizing the total of two tanks change with reaction order?

2. The Optimal Split

Computation 2.1: optimal intermediate concentration and residence times of the two tanks.

order, remainder	intermediate	first	second	first/second	total
0.5, 0.1	0.358698	1.070776	0.818074	1.308899	1.888850
0.5, 0.01	0.148878	2.205858	1.388775	1.588347	3.594633
1, 0.1	0.316228	2.162278	2.162278	1.000000	4.324555
1, 0.01	0.100000	9.000000	9.000000	1.000000	18.000000
2, 0.1	0.259170	11.029288	15.917041	0.692923	26.946329
2, 0.01	0.057910	280.917867	479.103817	0.586340	760.021685

At the optimum, the slope of the total was at most 3.73×10^{-4} in size for the six cases. At first order the equal tank matched the 2.162278 and 9.000000 of Paper 960 within 10^{-5} .

Observation 2.2

whether the first tank is smaller or larger reverses across order 1. At order 2 the first tank was smaller for both remainders, at order 0.5 it was larger, and at order 1 the two were equal: going from remainder 0.1 to 0.01, the size ratio moved away from 1 (from 0.692923 to 0.586340 at order 2, and from 1.308899 to 1.588347 at order 0.5).

3. Against Equal Sizes

Computation 3.1: equal tank, equal-size total, and the ratio of optimal to equal total.

order, remainder	one equal tank	equal-size total	optimal/equal
0.5, 0.1	0.948683	1.897367	0.995511
0.5, 0.01	1.834076	3.668152	0.979957
1, 0.1	2.162278	4.324555	1.000000
1, 0.01	9.000000	18.000000	1.000000
2, 0.1	13.650190	27.300380	0.987031
2, 0.01	392.299140	784.598279	0.968676

Observation 3.2

the gain from unequal sizes stays at a few percent. In all six cases the optimum was smaller than or equal to equal sizes, and the largest gain was 0.968676 at order 2 and remainder 0.01: the smaller the remainder, the larger the gain.

4. The Separator

Observation 4.1

the separator is whether the reaction rate changes more steeply or more gently than the first power of concentration. Halving concentration quarters the rate at order 2, multiplies it by 0.707107 at order 0.5, and halves it at order 1. At the optimum, the first tank was small at order 2 (intermediate concentration 0.259170) and large at order 0.5 (0.358698): at order 1 the intermediate concentration was 0.316228 and the two tanks were the same size. Which tank comes first as the smaller switched where the order crosses 1.

Observation 4.2

what is not said. Three or more tanks, tanks at different temperatures, combination with plug flow, cost, real reactions or equipment, and which split to use are not treated. What is shown is, for set orders and remainders, the optimal size split of two tanks and its gain over equal sizes.

5. Roll Call and Closing

1. **balance equation of a stirred tank** (Section 1): **the intermediate concentration fixes both tanks.**
2. **equal sizes at first order** (Computation 2.1): **matches Paper 960.**
3. **first/second 0.692923 at order 2** (Computation 2.1): **smaller tank first.**
4. **1.308899 at order 0.5** (Computation 2.1): **larger tank first.**
5. **gain at most 0.968676** (Computation 3.1): **a few percent.**
6. **steeper or gentler than the first power** (Observation 4.1): **the separator; the scope notes of Papers 960 and 931.**

Closing: The smallest pair of tanks puts the smaller tank first for a second-order reaction and the larger first for order one half. At remainder 0.1, the optimal first/second ratio was 0.692923 at order 2, 1.308899 at order 0.5, and equal at order 1: the gain was a few percent, down to 0.968676 of the equal-size total at order 2 and remainder 0.01. The separator is whether the reaction rate changes more steeply or more gently than the first power of concentration. Placed among the earlier papers: to the tanks in series that Papers 960 and 931 counted with first-order reaction and equal sizes, the second-order reactions and size splits they did not include were added. To be honest. There are only two tanks, with neither cost nor temperature included. How the direction of the optimal split changes with order, and no further.

References

1. Fogler, H. S. (2006). *Elements of Chemical Reaction Engineering*. Prentice Hall.
2. Levenspiel, O. (1999). *Chemical Reaction Engineering*. Wiley.
3. Yamagishi, Y. (2026). *With the Same Mean Residence Time, Splitting a Mixed Vessel into More Tanks in Series Lets Less Leave Early and Brings Reaction Close to Plug Flow* [Paper 931]. YOI: <https://www.yamagishiyuuki.com/yoi/931>
4. Yamagishi, Y. (2026). *The Cheapest Number of Tanks in Series Scales with the Decades to Remove, and Each Tank's Share Is Set by the Cost Exponent Alone* [Paper 960]. YOI: <https://www.yamagishiyuuki.com/yoi/960>

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)