

In a Solution of Membrane-Permeant Solute, a Cell Never Shrinks and Keeps Swelling Even at Equal Osmolality

— adding a permeant solute to saline makes the cell shrink once and return, and both the swelling rate and the depth of shrinkage are set by how easily the solute crosses —the separator is whether the solute crosses the membrane or not — [Paper 973]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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YOI: <https://www.yamagishiyuuki.com/yoi/973>

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Abstract

A cell whose membrane passes only water settles faster when shrinking than when swelling for the same factor of concentration difference (Paper 966). There, solutes that cross the membrane, such as urea or glucose, were not included. This paper adds a membrane-permeant solute to the cell of Paper 966 (osmotically inactive fraction 0.4) and, for each ratio ω of solute to water permeability (a set value), counts the volume in a solution of 290 mOsm permeant solute alone and in 290 mOsm saline with 290 mOsm permeant solute added. No new theorem or law is claimed. First, in permeant solute alone, the cell never shrinks and keeps swelling: for all four ω (0.1, 1, 10 and 1000), the smallest volume was 1.000000 (Section 2). Second, the swelling rate is set by how easily the solute crosses: the time to reach 1.6 times the volume (a set mark) was **10.4680**, **2.1330**, **1.0468** and **0.9015** for $\omega = 0.1, 1, 10$ and **1000**, approaching the closed-form limit for instant solute equilibration, **0.900000** (Section 2). Third, adding a permeant solute to saline makes the cell shrink once and return: the smallest volume was **0.732948**, **0.835040**, **0.956697** and **0.999405** for $\omega = 0.1, 1, 10$ and **1000**, reached at times **0.7286**, **0.4114**, **0.1398** and **0.0041**, and all returned to 1 in the end (**0.998592** at time 60 for $\omega = 0.1$) (Section 3). Fourth, the less permeant the solute, the closer the shrinkage comes to that of an impermeant solute: the smaller ω , the deeper the shrinkage, approaching the settled volume **0.700000** for 580 mOsm of impermeant solute alone (Paper 966) (Section 3). Fifth, the separator is whether the solute crosses the membrane or not: counted by osmolality, permeant and impermeant solutes are counted alike, but a permeant solute equalizes inside and outside over time and loses its power to move water. The time it takes to lose that power set both the swelling rate and the depth of shrinkage (Section 4). **Keywords:** permeant solute, cell volume, osmotic pressure, urea, membrane transport, hemolysis.

Scope of this paper (scope note): No new theorem or law is claimed: that a permeant solute equalizes inside and outside over time and loses its power to move water, that cells may swell in a solution of permeant solute even at equal osmolality, the equations describing volume with two permeabilities for water and solute, and measuring solute permeability from a shrink-swell response, are all known in physiology and membrane transport. These are set values: the osmotically inactive fraction 0.4 and the time unit are as in Paper 966, and the permeability ratios ω (0.1, 1, 10, 1000) and the lysis mark of 1.6 times the volume are set here. No value for any solute or cell is claimed. Solute-water coupling is not included: solute dragged by water flow (the reflection coefficient), membrane tension, volume regulation by the cell and lysis itself are not included, and membrane area is held constant. Medical use is not discussed, which solution to use when is not treated. Relation to earlier papers: Paper 966, in its scope note, took only water to cross the membrane, leaving out permeant solutes such as urea or glucose, the volume at lysis, membrane tension and volume regulation; this paper sits in the seat of permeant solutes (no earlier occupant). Paper 966 showed that with a membrane passing only water, a cell settles faster when shrinking than when swelling for the same factor of concentration difference. Here a permeant solute is added to the same cell, and volume is counted as moving even in solutions of equal osmolality. Paper 922 counted the osmolality of normal saline and compared urea and glucose on the premise that they cross the membrane. Here that premise is counted as motion in time. What is added: finding for four ω the time for volume to keep swelling in permeant solute alone, comparing with the closed-form limit, finding the depth and time of the single shrinkage when permeant solute is added to saline, showing that the depth approaches the impermeant value, and placing the separator on “crosses the membrane, or not”.

1. Notation and the Question

Setting: as in Paper 966, volume is the ratio x to the original, with osmotically inactive fraction $b = 0.4$ and constant membrane area. With impermeant solute inside $1 - b$ (in units of 290 mOsm times original volume), permeant solute inside m , and impermeant r_i and permeant r_p outside (in units of 290 mOsm),

$$\frac{dx}{dt} = \frac{1 - b + m}{x - b} - (r_i + r_p), \quad \frac{dm}{dt} = \omega \left(r_p - \frac{m}{x - b} \right)$$

Here ω is solute permeability divided by water permeability, set to 0.1, 1, 10 and 1000. The two solutions are permeant solute alone ($r_i = 0, r_p = 1$) and saline with permeant solute added ($r_i = 1, r_p = 1$). In the limit of instant solute equilibration $m = r_p(x - b)$, and in permeant solute alone $(x - b)^2 = (1 - b)^2 + 2(1 - b)t$.

Question: with a membrane-permeant solute present, how does cell volume move even in solutions of equal osmolality?

2. Permeant Solute Alone

Computation 2.1: a solution of 290 mOsm permeant solute alone.

ω	smallest volume	time to reach 1.6
0.1	1.000000	10.4680
1	1.000000	2.1330
10	1.000000	1.0468
1000	1.000000	0.9015

In the instant-equilibration limit, the closed form gives 0.900000 to reach 1.6, within 0.01 of 0.9015 for $\omega = 1000$.

Observation 2.2

in permeant solute alone, the cell never shrinks and keeps swelling. At first inside and outside osmolality are equal and water does not move, but as solute enters, the inside becomes more concentrated and water keeps coming in: the swelling rate rose with ω : reaching 1.6 took 10.4680 at $\omega = 0.1$ and 0.9015, close to the limit, at $\omega = 1000$.

3. Saline with Permeant Solute Added

Computation 3.1: 290 mOsm saline with 290 mOsm permeant solute added.

ω	smallest volume	time	final volume
0.1	0.732948	0.7286	0.998592
1	0.835040	0.4114	1.000000
10	0.956697	0.1398	1.000000
1000	0.999405	0.0041	1.000000

The final volume is the value at time 60 for $\omega = 0.1$, 1 and 10 and at time 5 for $\omega = 1000$. The settled volume for 580 mOsm of impermeant solute alone in Paper 966 is 0.700000.

Observation 3.2

adding permeant solute to saline makes the cell shrink once and return. At first the outside acts like 580 mOsm and water leaves, and as solute enters, water comes back: for all four ω , the larger ω , the shallower and earlier the minimum; at $\omega = 0.1$ the cell shrank to 0.732948, close to the impermeant 0.700000.

4. The Separator

Observation 4.1

the separator is whether the solute crosses the membrane or not. Counted by osmolality, permeant and impermeant solutes are both counted as 290 mOsm: a permeant solute loses its power to move water as it equalizes, while an impermeant one keeps it. In permeant solute alone, the inside impermeant solute keeps drawing water against a powerless outside; with saline added, the cell shrank only until the permeant solute lost its power. That time was set by ω .

Observation 4.2

what is not said. Real permeabilities of urea or glucose, reflection coefficients, membrane tension, volume regulation, lysis itself and medical use are not treated. What is shown is that, in a model with two permeabilities for water and solute, cell volume moves even at equal osmolality, with rate and depth set by permeability.

5. Roll Call and Closing

1. two equations for water and solute (Section 1): permeant solute added to the cell of Paper 966.
2. smallest volume 1.000000 in permeant solute alone (Computation 2.1): never shrinks, keeps swelling.
3. time to 1.6 from 10.4680 to 0.9015 (Computation 2.1): approaches the limit 0.900000.
4. with saline, smallest volume 0.732948 to 0.999405 (Computation 3.1): shrinks once and returns.
5. closer to 0.700000 for smaller ω (Computation 3.1): approaches the impermeant case.
6. crosses the membrane, or not (Observation 4.1): the separator; the scope note of Paper 966.

Closing: In a solution of membrane-permeant solute, a cell never shrinks and keeps swelling even at equal osmolality. Adding a permeant solute to the cell of Paper 966, in 290 mOsm of permeant solute alone the smallest volume was 1.000000 for all four ω , and reaching 1.6 took 10.4680 at $\omega = 0.1$ and 0.9015 at 1000: with the solute added to saline the cell shrank once, to 0.732948 at $\omega = 0.1$ and 0.999405 at 1000, and returned to its original volume. The separator is whether the solute crosses the membrane or not. Placed among the earlier papers: to the volume motion Paper 966 counted with a water-only membrane, the permeant solutes Paper 966 did not include were added, and the permeant-solute premise of Paper 922 was counted as motion in time. To be honest: neither reflection coefficients nor volume regulation are included. With two permeabilities alone, how volume moves in solutions of equal osmolality, and no further.

References

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Yuuki YAMAGISHI (山岸 勇輝)

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