

Near Equal Break-Even Food Levels the Loser Lingers, and Exactly There the Ratio Stops

— the time to disappear is nearly inversely proportional to the difference in flow rate, with coefficients of 3.4040 and 6.1650 depending on the winner —the separator is whether the flow rate is above or below the equal point, or on it — [Paper 975]

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September 2026

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Abstract

Of two species competing for one food in a flow-through culture, only the one breaking even on less food (the smaller R^*) remains, and the winner switches with the flow rate (Paper 953). What happens exactly where the two R^* values are equal was not treated there. Using the two species of Paper 953, this paper brings the flow rate toward 0.233333 per hour, where the R^* values are equal, and counts the time until the loser falls below one thousandth of the winner and the ratio that remains exactly at the equal point. No new theorem or law is claimed. First, above the equal point A wins, below it B wins: A remained at the 4 points with flow-rate differences +0.030 to +0.001, and B at the 4 points from -0.001 to -0.030 (Section 2). Second, closer to the equal point, the loser lingers longer: the time to fall below one thousandth was 337.5 and 3404.0 hours at differences +0.010 and +0.001, and 646.5 and 6165.0 hours at -0.010 and -0.001 (Section 2). Third, the time is nearly inversely proportional to the difference, with a coefficient that depends on the winner: time times difference was 3.3750, 3.3975 and 3.4040 on the side where A wins and 6.4650, 6.2310 and 6.1650 where B wins (differences 0.01, 0.003, 0.001) (Section 2). Fourth, exactly at the equal point, the loser does not disappear and the ratio stops: food settled at 0.700000 g/L, and A's share was 0.982561, 0.962918 and 0.996716 from initial shares 0.5, 0.1 and 0.9, the same within 10^{-6} at 10000 and 20000 hours (Section 3). Fifth, the separator is whether the flow rate is above or below the equal point, or on it: above or below, the winner is settled and the difference changes only the time to disappear. On the point, no winner is settled, and the remaining ratio is set by the initial ratio and the growth along the way, differing for all three initial ratios (Section 4). **Keywords:** resource competition, chemostat, coexistence, competitive exclusion, Monod equation, break-even resource level.

Scope of this paper (scope note): No new theorem or law is claimed: the Monod equation, the balance of a flow-through culture, that only the species with smaller R^* remains, and that when the R^* values are exactly equal two species can remain together on a line of balances, are all known in resource competition theory. Values are those of Paper 953: the maximum growth rates, half-saturation concentrations and yields of the two species, the incoming food of 10.0 g/L and the initial amounts are the values of Paper 953, corresponding to no alga or microbe. Chance is not included: the equations are deterministic; chance loss at small numbers, mutation and fluctuating flow rate are not included. Coexistence is not recommended: how to keep two species together in a real culture is not treated. Relation to earlier papers: Paper 953, in its scope note, did not discuss coexistence, examining only four flow rates and leaving out what happens exactly where the break-even food levels are equal; this paper sits in that seat (no earlier occupant). Paper 953 showed that what remains in a flow-through culture is not the fastest-growing species but the one breaking even on less food, with the winner switching at a flow rate of 0.233333 per hour. Here the neighborhood of that switching point and the point itself are counted. What is added: finding the time to disappear at 8 points on both sides of the equal point, showing that the time is nearly inversely proportional to the difference with a coefficient depending on the side, confirming at two times that the ratio stops at the equal point, showing that the remaining ratio differs with the initial ratio, and placing the separator on “above or below, or on the point”.

1. Notation and the Question

Setting: as in Paper 953, with food S , species amounts x_A and x_B , flow rate D , incoming food $S_0 = 10.0$ g/L, growth $\mu_i(S) = \mu_{\max,i} S / (K_i + S)$ (A 0.40 per hour and 0.50 g/L, B 0.25 per hour and 0.05 g/L) and yield 0.5:

$$\frac{dS}{dt} = D(S_0 - S) - \frac{\mu_A x_A + \mu_B x_B}{0.5}, \quad \frac{dx_i}{dt} = (\mu_i(S) - D) x_i$$

The break-even food level $R_i^* = K_i D / (\mu_{\max,i} - D)$ is equal for the two species at $D_c = 0.233333$ per hour ($R^* = 0.700000$ g/L). The system was followed in steps of 0.5 hours up to 200000 hours (differing by 0.04 hours from steps of 0.02 at $D = 0.20$), and the time when the loser fell below one thousandth of the winner was recorded.

Question: near and at the point of equal R^* , when does the loser disappear, and what remains?

2. Near the Equal Point

Computation 2.1: flow rate D_c plus a difference, and the time to fall below one thousandth.

difference	flow rate (per hour)	winner	time below one thousandth (hours)
+0.030	0.263333	A	111.0
+0.010	0.243333	A	337.5
+0.003	0.236333	A	1132.5
+0.001	0.234333	A	3404.0
−0.001	0.232333	B	6165.0
−0.003	0.230333	B	2077.0
−0.010	0.223333	B	646.5
−0.030	0.203333	B	240.5

Time times difference was 3.3750, 3.3975 and 3.4040 where A wins (differences 0.01, 0.003, 0.001) and 6.4650, 6.2310 and 6.1650 where B wins. On both sides, reducing the difference tenfold made the time more than 5 times longer.

Observation 2.2

closer to the equal point, the loser lingers longer. The time grew nearly in inverse proportion to the difference, taking 3404.0 hours where A wins and 6165.0 hours where B wins at a difference of 0.001: time times difference differed between the two sides (3.4040 and 6.1650), and for the same difference the loser disappeared more slowly where B wins.

3. Exactly at the Equal Point

Computation 3.1: A's share at $D = D_c$ for three initial amounts.

initial (A, B)	initial A share	after 10000 hours	after 20000 hours (A, B amounts)
0.05, 0.05	0.500000	0.982561	0.982561 (4.568907, 0.081093)
0.01, 0.09	0.100000	0.962918	0.962918 (4.477568, 0.172432)
0.09, 0.01	0.900000	0.996716	0.996716 (4.634730, 0.015270)

In all three, food settled at 0.700000 g/L, and A's share at 10000 and 20000 hours agreed within 10^{-6} . The three remaining shares differed from each other by more than 0.01, and all exceeded the initial share.

Observation 3.2

exactly at the equal point, the loser does not disappear and the ratio is set along the way and stops. Once food settles at R^* , both species grow at the flow rate and the ratio no longer moves: the remaining ratio was not the initial ratio: A's share grew from every initial ratio.

4. The Separator

Observation 4.1

the separator is whether the flow rate is above or below the equal point, or on it. Above or below, the winner was set by the sign of the difference, and the size of the difference changed only the time to disappear: on the point, no winner was set, and three initial ratios left three different ratios (0.962918, 0.982561, 0.996716).

Observation 4.2

what is not said. Chance loss at small numbers, mutation, fluctuating flow rate, and how to keep species together in a real culture are not treated. What is shown is that in the deterministic equations of Paper 953, the time to disappear grows inversely with the difference near the equal point, and the ratio stops on the point.

5. Roll Call and Closing

1. the equations of Paper 953 and $D_c = 0.233333$ per hour (Section 1): the equal- R^* point.
2. 3404.0 and 6165.0 hours at a difference of 0.001 (Computation 2.1): the loser lingers.
3. time times difference **3.4040** and 6.1650 (Computation 2.1): inverse proportion, coefficient by side.
4. A's share stops at **0.982561** on the point (Computation 3.1): no disappearance.
5. three initial ratios, three remaining ratios (Computation 3.1): set by start and path.
6. above or below, or on the point (Observation 4.1): the separator; the scope note of Paper 953.

Closing: Near equal break-even food levels the loser lingers, and exactly there the ratio stops. For the two species of Paper 953, shifting the flow rate by 0.001 from the equal- R^* value of 0.233333 per hour made the loser take 3404.0 hours (A winning) and 6165.0 hours (B winning) to fall below one thousandth, with time times difference 3.4040 and 6.1650: exactly at the equal point food settled at 0.700000 g/L, and A's share rose from 0.5 to 0.982561 and stopped there. The separator is whether the flow rate is above or below the equal point, or on it. Placed among the earlier papers: of the winner switching that Paper 953 counted at four flow rates, the neighborhood of the switching point and the point itself, which Paper 953 did not treat, were counted. To be honest. These equations leave out chance loss, and in a real culture a scarce species can vanish by chance. What happens near and on the point in deterministic equations, and no further.

References

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