

The Smaller a Solute's Reflection Coefficient, the Shallower a Cell Shrinks and the Faster It Swells in That Solute Alone

— with solute carried by the water flow, the brief shrinkage in a solution of equal osmolality is not reduced in proportion to the reflection coefficient, and at coefficient 0 the cell does not shrink and swells as fast as in water — the separator is whether the solute crosses by a path apart from water or rides the water flow — [Paper 986]

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— *From thinking to syncing.* —

Abstract

In a solution of membrane-permeant solute, a cell never shrinks and keeps swelling even at equal osmolality (Paper 973). The share of solute pulled along by water flow (the reflection coefficient) was not included there, and the solute was taken not to use the water path. Adding a reflection coefficient σ and the solute carried by water flow to the volume equations of Paper 973, this paper finds how volume moves in saline with added permeant solute and in a solution of permeant solute alone. No new theorem or law is claimed. First, the smaller the reflection coefficient, the shallower the brief shrinkage: in saline with permeant solute and permeability ratio $\omega=0.1$, the minimum volume for $\sigma=1, 0.9, 0.75, 0.5, 0.25$ and 0 was **0.732948**, 0.743511, 0.764875, 0.817668, 0.895605 and 1.000000. $\sigma=1$ is the value of Paper 973 (Section 2). Second, the shrinkage is deeper than proportional to the reflection coefficient: dividing the depth of shrinkage by that at $\sigma=1$ gave, at $\sigma=0.5$, **0.682758**, 0.598414 and 0.523943 for $\omega=0.1, 1$ and 10 , larger than σ in all 12 combinations (Section 2). Third, in permeant solute alone, the smaller the reflection coefficient, the faster the swelling: the time to reach 1.6 times the volume fell from 10.4680 at $\sigma=1$ to **0.9000** at $\sigma=0$ for $\omega=0.1$, and from 1.0470 to 0.9000 for $\omega=10$ (Section 3). Fourth, a solute with reflection coefficient 0 was as if absent: at $\sigma=0$, for all three ω , the cell did not shrink in saline with permeant solute (minimum 1.000000) and reached 1.6 times within 2×10^{-3} of the water formula's 0.9 in permeant solute alone. In saline without permeant solute (solved at $\sigma=0.5, \omega=1$) the volume did not move (Section 4). Fifth, the separator is whether the solute crosses by a path apart from water or rides the water flow: at $\sigma=1$ the solute carries its full osmotic pull on water; at $\sigma=0$ it crosses with water by the same path and pulls none. In between, the depth ratio did not equal σ , and was closer to σ the larger ω (Section 5). **Keywords:** reflection coefficient, osmosis, cell volume, membrane transport, solute drag, osmolality.

Scope of this paper (scope note): No new theorem or law is claimed: expressing the osmotic effect of a permeant solute by a reflection coefficient, the membrane flux equations including solute carried by water flow (Kedem and Katchalsky), and that a solute with reflection coefficient 0 draws no water osmotically, are known in the theory of membrane transport. These are set values: the non-osmotic fraction $b = 0.4$, the solutions and permeability ratios $\omega = 0.1, 1$ and 10 are model values of Paper 973, and reflection coefficients from 0 to 1 are set here. They correspond to no cell, membrane or solute. The equations are simplified: the concentration of solute carried by water is taken as the arithmetic mean of inside and outside; membrane tension, volume regulation, haemolysis and change of area are not included. No medical use is stated, which solution to use when is not treated. Relation to earlier papers: Paper 973, in its scope note, did not include coupling between solute and water, leaving out the share of solute pulled by water flow (the reflection coefficient), membrane tension, volume regulation and haemolysis itself; this paper sits in the seat of the reflection coefficient (no earlier occupant). Paper 973 showed that in a solution of membrane-permeant solute a cell never shrinks and keeps swelling even at equal osmolality. Here the change in shrinkage and swelling when that solute is pulled across by water flow is counted. What is added: finding minimum volumes and times to 1.6 times for three permeability ratios and six reflection coefficients, reproducing Paper 973 at coefficient 1, measuring that shrinkage is deeper than proportional to the coefficient, confirming that coefficient 0 behaves as water, and placing the separator on “a path apart from water, or riding the water flow”.

1. Notation and the Question

Setting: as in Paper 973, volume relative to the original x , non-osmotic fraction $b = 0.4$, impermeant solute inside $1 - b$, permeant solute inside m , impermeant solute outside r_i and permeant solute outside r_p (in units of 290 mOsm). With a reflection coefficient σ and the concentration of solute carried by water taken as the arithmetic mean of inside and outside,

$$\frac{dx}{dt} = \frac{1-b}{x-b} - r_i + \sigma \left(\frac{m}{x-b} - r_p \right), \quad \frac{dm}{dt} = \omega \left(r_p - \frac{m}{x-b} \right) + (1-\sigma) \frac{1}{2} \left(\frac{m}{x-b} + r_p \right) \frac{dx}{dt}$$

which returns to Paper 973 at $\sigma = 1$. With $\omega = 0.1, 1, 10$ and $\sigma = 1, 0.9, 0.75, 0.5, 0.25, 0$, the equations were solved by fourth-order Runge-Kutta with step 0.001 (halving the step at $\omega = 1, \sigma = 0.5$ left the minimum volume 0.901286 unchanged). The solutions are saline with permeant solute ($r_i = 1, r_p = 1$, to time 60) and permeant solute alone ($r_i = 0, r_p = 1$, until 1.6 times, to time 200).

Question: when solute is pulled across the membrane by water flow, how does cell volume move in solutions of equal osmolality?

2. Shrinkage in Saline with Permeant Solute

Computation 2.1: minimum volume (time in parentheses).

σ	$\omega = 0.1$	$\omega = 1$	$\omega = 10$
1	0.732948 (0.7290)	0.835040 (0.4110)	0.956697 (0.1400)
0.9	0.743511 (0.7760)	0.845489 (0.4280)	0.960593 (0.1410)
0.75	0.764875 (0.8580)	0.863759 (0.4540)	0.966663 (0.1440)
0.5	0.817668 (1.0320)	0.901286 (0.5010)	0.977312 (0.1470)
0.25	0.895605 (1.2590)	0.947255 (0.5510)	0.988487 (0.1510)
0	1.000000	1.000000	1.000000

$\sigma = 1, \omega = 0.1$ reproduced the minimum 0.732948 of Paper 973. At $\omega = 0.1$ the volume at time 60 was 0.998592 for $\sigma = 1$ and 0.999989 for $\sigma = 0.25$.

Computation 2.2: depth of shrinkage (1– minimum volume) divided by the depth at $\sigma = 1$.

σ	$\omega = 0.1$	$\omega = 1$	$\omega = 10$
0.9	0.960448	0.936659	0.910031
0.75	0.880446	0.825901	0.769856
0.5	0.682758	0.598414	0.523943
0.25	0.390917	0.319746	0.265882
0	0.000000	0.000000	0.000000

Observation 2.3

the shrinkage is deeper than proportional to the reflection coefficient, and closer to proportional the larger ω . In all 12 combinations of three ω and four σ (0.9 to 0.25), the depth ratio was larger than σ : at $\sigma = 0.5$ it fell from 0.682758 at $\omega = 0.1$ to 0.523943 at $\omega = 10$. The time of the minimum was later the smaller σ .

3. Swelling in Permeant Solute Alone

Computation 3.1: time for the volume to reach 1.6 times.

σ	$\omega = 0.1$	$\omega = 1$	$\omega = 10$
1	10.4680	2.1330	1.0470
0.9	8.5110	1.9530	1.0260
0.75	5.7750	1.7090	0.9960
0.5	2.4180	1.3640	0.9550
0.25	1.3020	1.0960	0.9230
0	0.9000	0.9000	0.9000

Observation 3.2

the smaller the reflection coefficient, the faster the swelling. For all three ω , lowering σ shortened the time to 1.6 times: from 10.4680 to 0.9000 at $\omega = 0.1$ and from 1.0470 to 0.9000 at $\omega = 10$; the smaller ω , the larger the effect of σ .

4. Reflection Coefficient 0

Computation 4.1: at $\sigma = 0$, $dx/dt = (1 - b)/(x - b) - r_i$ and m drops out. In permeant solute alone ($r_i = 0$), $(x - b)^2 = (1 - b)^2 + 2(1 - b)t$ and the time to 1.6 times is 0.9. The solved times for three ω agreed with this within 2×10^{-3} , and in saline with permeant solute the minimum volume was 1. For comparison, saline without permeant solute ($\sigma = 0.5, \omega = 1$) gave a minimum volume of 1.000000000; the volume did not move.

Observation 4.2

a solute with reflection coefficient 0 was, for cell volume, as if absent. Saline with permeant solute behaved as saline, and permeant solute alone as water: the solute still crosses into the cell, but draws no water.

5. The Separator

Observation 5.1

the separator is whether the solute crosses by a path apart from water or rides the water flow. At $\sigma = 1$ the solute does not use the water path and the whole concentration difference draws water. At $\sigma = 0$ the solute crosses with water by the same path and draws none: in between, the depth ratio did not equal σ (0.523943 to 0.682758 at $\sigma = 0.5$) and was closer to σ the larger ω . Why the depth ratio exceeds σ was not measured term by term here.

Observation 5.2

what is not said. Reflection coefficients of real membranes or solutes, other choices for the concentration carried by water (such as a logarithmic mean), membrane tension, volume regulation, haemolysis and medical use are not treated. What is shown is how adding a reflection coefficient to the model of Paper 973 changes the brief shrinkage and the speed of swelling.

6. Roll Call and Closing

1. equations with reflection coefficient and solute drag (Section 1): return to Paper 973 at $\sigma = 1$.
2. minimum volume **0.732948** to 1.000000 (Computation 2.1): shallower for smaller coefficient.
3. depth ratio **0.682758** ($\sigma = 0.5$) (Computation 2.2): deeper than proportional.
4. time to 1.6 times 10.4680 to **0.9000** (Computation 3.1): faster swelling.
5. $\sigma = 0$ matches the water formula (Computation 4.1): the solute is as if absent.
6. apart from water, or riding the water flow (Observation 5.1): the separator; the scope note of Paper 973.

Closing: The smaller a solute's reflection coefficient, the shallower a cell shrinks and the faster it swells in that solute alone. Adding a reflection coefficient and solute drag to the volume equations of Paper 973, the minimum volume in saline with permeant solute at $\omega=0.1$ rose from 0.732948 at $\sigma=1$ to 1.000000 at $\sigma=0$, and the depth ratio exceeded σ in all 12 combinations: the time to 1.6 times in permeant solute alone fell from 10.4680 to 0.9000, matching the water formula at $\sigma=0$. The separator is whether the solute crosses by a path apart from water or rides the water flow. Placed among the earlier papers: to the volume changes Paper 973 counted with the solute kept off the water path, the reflection coefficient Paper 973 did not include was added. To be honest. This counts volume changes in one set of

equations with the carried concentration taken as an arithmetic mean. How the reflection coefficient changes shrinkage and swelling, and no further.

References

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