

Stacking a Transposition Table on Alpha-Beta in Tic-Tac-Toe Does Not Multiply the Savings

— a transposition table cuts minimax expansions down to the reachable positions, but added to alpha-beta its effect weakens, and the combined factor is far larger than the product of the two factors —the separator is whether the table holds the value of a position itself or upper and lower bounds cut by the window — [Paper 989]

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Abstract

The number of leaves alpha-beta search reads ranges, with the order of children, from over eight tenths of all to the order of their square root (Paper 926). A transposition table, which keeps the values of positions for reuse, was not treated there. Starting from the empty board of tic-tac-toe, this paper adds a transposition table to minimax and to alpha-beta and counts the positions expanded, for two move orders. No new theorem or law is claimed. First, a transposition table cuts minimax expansions to the number of reachable positions: minimax expanded 549946 positions, and with a transposition table 5478 (0.009961 times), equal to the number of positions reachable from the empty board (Section 2). Second, added to alpha-beta, the transposition table helps less: alpha-beta with moves in index order expanded 18297 positions, and with a transposition table 2700, a factor of 0.147565. With centre and corners first it went from 7275 to 1242 (0.170722) (Section 3). Third, the combined factor is far larger than the product of the two: in index order the combined factor 0.004910 was 14.814326 times the product 0.000331 of the table alone (0.009961) and alpha-beta alone (0.033271). With centre and corners first it was 17.139045 times (Section 3). Fourth, most values left in the alpha-beta table are not exact: in the alpha-beta table with rotations and reflections merged, the share of positions with an exact value was 0.294821 in index order and 0.338235 with centre and corners first; the rest were upper or lower bounds cut by the window (Section 4). Fifth, the separator is whether the table holds the value of a position itself or upper and lower bounds cut by the window: the minimax table held only values themselves, and a second meeting was always settled from it. The alpha-beta table held bounds per window, and the factor of alpha-beta with a table over alpha-beta without (0.147565) did not reach the factor for minimax (0.009961) (Section 5). **Keywords:** transposition table, alpha-beta search, minimax, tic-tac-toe, search, pruning.

Scope of this paper (scope note): No new theorem or law is claimed: minimax, alpha-beta pruning, transposition tables that keep position values, storing values with upper- and lower-bound flags under alpha-beta, and that tic-tac-toe is a draw, are all known in search. A small game only: only reading tic-tac-toe from the empty board to the end is treated; depth limits, evaluation functions, games on larger boards and the speed of real programs are not. Two move orders only: only index order and centre-and-corners-first are compared, and re-ordering by shallow searches (Paper 951) is not combined. Reasons are not separated: why the two savings do not multiply is not measured term by term. Only expanded positions and the kinds of values left in the table are set out. Relation to earlier papers: Paper 926, in its scope note, did not treat real game programs, leaving out evaluation functions, iterative deepening, transposition tables and move-ordering heuristics themselves, and Paper 951, in its scope note, also did not include other heuristics, leaving out reuse of identical positions (transposition tables); this paper sits in the seat of transposition tables (no earlier occupant; Paper 951 sat in the iterative-deepening and move-ordering seat of the scope note of Paper 926). Paper 607 showed that how often the first player wins at tic-tac-toe splits three ways by what is taken as equal, counting 5478 reachable positions (765 merged) and that the best-play value is a draw. Here the same set of positions is counted by how much search expands. What is added: counting expanded positions for six ways of reading and two move orders, confirming that the table size equals the number of reachable positions, comparing the product of the two savings with the combined factor, giving the share of exact values in the alpha-beta table, and placing the separator on “the value itself, or upper and lower bounds”.

1. Notation and the Question

Game and values: from the empty board of tic-tac-toe, the value for the side to move (win 1, draw 0, loss -1) is found by reading to the end.

Six ways of reading: minimax (read everything); minimax with a transposition table (keyed by the board, settling a second meeting from the table); the same with the 8 rotations and reflections merged in the key; alpha-beta (cutting branches with a window $[\alpha, \beta]$); alpha-beta with a transposition table (keeping a value read as an upper bound if below the window, a lower bound if above, exact if inside, narrowing the window by stored bounds and settling from the table when the window closes); and the same with the 8 symmetries merged. Two move orders: index order (top left to bottom right) and centre, corners, edges. Expanded positions are those entered to examine children rather than settled from the table.

Question: how much does a transposition table cut expansions under minimax and under alpha-beta, and do the savings multiply when stacked?

2. Minimax and the Transposition Table

Computation 2.1: 5478 positions are reachable from the empty board, 765 with rotations and reflections merged (as in Paper 607).

reading	expanded positions	settled from table	table size
minimax	549946	0	none
minimax with table	5478	10690	5478
same, symmetries merged	765	1506	765

The minimax count was the same for both move orders. The table size equalled the number of reachable positions, and the merged table the merged number. All values were 0.

3. Alpha-Beta and the Transposition Table

Computation 3.1: expanded positions (settled from table in parentheses).

reading	index order	centre and corners first
alpha-beta	18297	7275
alpha-beta with table	2700 (2077)	1242 (739)
same, symmetries merged	599 (486)	285 (187)

Computation 3.2: comparison of factors.

factor	index order	centre and corners first
table (under minimax)	0.009961	0.009961
table (under alpha-beta)	0.147565	0.170722
alpha-beta (without table)	0.033271	0.013229
alpha-beta (with table)	0.492881	0.226725
both (against minimax)	0.004910	0.002258
product of table and alpha-beta factors	0.000331	0.000132
both over product	14.814326	17.139045

Observation 3.3

the two savings do not multiply. The table cut minimax to 0.009961, but alpha-beta only to 0.147565 (0.170722 with centre and corners first): conversely, alpha-beta cut 0.033271 without a table but only 0.492881 with one. In both move orders the combined factor exceeded ten times the product.

4. Values Left in the Table

Computation 4.1: the alpha-beta table with rotations and reflections merged had 502 entries in index order and 272 with centre and corners first, of which 148 (0.294821) and 92 (0.338235) had exact values. Expanded positions over reachable positions were 0.492881 for alpha-beta with table in index order, and 0.783007 with symmetries merged.

Observation 4.2

much of the alpha-beta table held bounds cut by the window. Only exact values can be used with any window met next: an upper or lower bound was usable only when the next window lay beyond that bound; otherwise the position was expanded again.

5. The Separator

Observation 5.1

the separator is whether the table holds the value of a position itself or upper and lower bounds cut by the window. Minimax has no window, so its table held only values themselves, and a second meeting was always settled (10690 times settled from the table): alpha-beta cuts outside the window, so much of its table held bounds, and the table's effect stopped at 0.147565. Why the two savings do not multiply was not measured term by term.

Observation 5.2

what is not said. Depth limits, evaluation functions, games on larger boards, combination with re-ordering by shallow searches, table size limits and replacement, and the speed of real programs are not treated. What is shown is what happens to the savings of a transposition table and alpha-beta when stacked, in a small game that can be read to the end.

6. Roll Call and Closing

1. six ways of reading and two move orders (Section 1): all values are draws.
2. the table cuts minimax to **0.009961** (Computation 2.1): down to the reachable positions.
3. added to alpha-beta, **0.147565** (Computation 3.2): a weaker effect.
4. both over product **14.814326** (Computation 3.2): no multiplication.
5. share of exact values **0.294821** (Computation 4.1): the rest are bounds.
6. value itself, or upper and lower bounds (Observation 5.1): the separator; the scope notes of Paper 926 and Paper 951.

Closing: Stacking a transposition table on alpha-beta in tic-tac-toe does not multiply the savings. Reading from the empty board to the end, the table cut minimax expansions from 549946 to 5478 (0.009961), but cut alpha-beta in index order only to 0.147565: the combined factor 0.004910 was 14.814326 times the product 0.000331 of the two, and the share of exact values in the alpha-beta table was 0.294821. The separator is whether the table holds the value of a position itself or upper and lower bounds cut by the window. Placed among the earlier papers: to the alpha-beta leaves Paper 926 and Paper 951 counted by move order, the transposition table neither treated was added, and the set of positions Paper 607 counted was counted from the side of search. To be honest. This counts expanded positions

in one small game that can be read to the end. What happens to the savings when a transposition table and alpha-beta are stacked, and no further.

References

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