

Reading the Interior from the Boundary — the Calderón Problem, Electrical Impedance Tomography (EIT), and the Inverse-Problem Stance — [Paper 99]

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— *From thinking to syncing.* —

Abstract

In Paper 89 (Zero-Tension and the Inverse Problem), the corpus raised the **inverse-problem stance** — reading the unseen "cause" from the observable "effect." This paper finds its **mathematical archetype** in the problem of Alberto Calderón. The **Calderón problem** (1980) asks: by applying voltages at the **surface (boundary)** of a body and measuring the outgoing currents, can one determine the **interior conductivity** $\gamma(\mathbf{x})$ (what lies where)? The boundary correspondence "voltage $\rightarrow$ current" is the **Dirichlet-to-Neumann map (DtN map)**. This is the mathematical foundation of **Electrical Impedance Tomography (EIT)** — imaging the interior without cutting — and Calderón himself reached the question from oil prospecting (reading the underground from surface measurements). The skeleton of the answer: **uniqueness is proven** (Sylvester–Uhlmann 1987 for $n \geq 3$; Nachman and others for $n = 2$) — the boundary DtN map determines the interior **uniquely**. But **honestly**: this inverse problem is severely **ill-posed**; a tiny measurement error at the boundary is greatly amplified in the interior (stability is **logarithmic**). Hence in practice **regularization** is required — inverse problems are powerful but unstable. The point of this paper is twofold: **(1)** to place the Calderón problem as the mathematical archetype of Paper 89's inverse-problem stance; **(2)** to sort the corpus's inverse problems — **inverse scattering** (Gel'fand–Levitan–Marchenko, Paper 89) and the **inverse boundary-value problem** (Calderón) — as **the same stance but different mathematics = distinct roots** (Papers 58/48). **Scope**: a Layer-2 structural interpretation of established inverse-problem theory; no new mathematics and no new prediction. The Universe OS is a Layer-3 non-science lens.

Positioning of this paper (discipline): This paper is a **structural interpretation** of established inverse-problem theory (the Calderón problem, the DtN map, EIT, the Sylvester–Uhlmann / Nachman uniqueness). It contains no new mathematics and no new prediction. The point is to give Paper 89's "inverse-problem stance" a mathematical archetype (Calderón), and to sort the corpus's inverse problems (scattering = GLM / boundary-value = Calderón) as **the same stance but distinct roots**. It **honestly records the ill-posedness (instability) of inverse problems** — even when unique, practice is perilous without regularization. "Universe OS" is a Layer-3 non-science lens, not a claim of mechanism.

1. Introduction: From Effect to Cause (the Inverse-Problem Stance)

In Paper 89 the corpus read a fishing master's "Zero-Tension" as the **inverse-problem stance** — drill a perfect "no-bite baseline (null)" into the body, and from **deviations** off it read the unseen "cause" underwater. From the observable **effect** (a faint deviation of the line, a scattered wave, a price move) recover the hidden **cause** (the fish, the potential, the state of the market) — this is the inverse problem. This paper finds the **mathematical archetype** of that stance in the problem of Alberto Calderón (1920–1998).

Calderón was an Argentine mathematician who, before academia, was an engineer at the oil company YPF. The prospecting question — **reading the underground structure from surface measurements alone** — led him to a monument of inverse problems. This paper builds no new theorem. It introduces the Calderón problem, shows it is the archetype of Paper 89's stance, and sorts the corpus's inverse problems as distinct roots.

2. The Calderón Problem: Can the Boundary DtN Map Determine the Interior?

Standard fact (the Calderón problem, 1980)

Let an unknown **conductivity** $\gamma(\mathbf{x}) > 0$ be distributed inside a region Ω (a human body, a geological layer). Applying a potential $\mathbf{f}$ at the boundary $\partial\Omega$ determines, inside, $\nabla \cdot (\gamma \nabla u) = 0$ (with $u|_{\partial\Omega} = \mathbf{f}$), and the outgoing current $\gamma \partial_\nu u|_{\partial\Omega}$ can be measured. This correspondence "voltage $\mathbf{f} \rightarrow$ current" is the **Dirichlet-to-Neumann map** Λ_γ (**DtN map**). Calderón's question: **from Λ_γ , measurable only at the boundary, can the interior $\gamma(\mathbf{x})$ be determined uniquely?** — this is the mathematical heart of **Electrical Impedance Tomography (EIT)**.

This is the pure form of an inverse problem. All one has is **boundary information** (an externally measurable effect). What one wants is the **interior distribution** (the unseen cause). The forward problem (given γ , compute the boundary current) is easy; the inverse (from the boundary current, recover γ) is vastly harder. It is exactly the stance of "from effect to cause."

3. Uniqueness: The Boundary Determines the Interior

Observation 3.1 (the uniqueness theorem)

The **uniqueness** of the Calderón problem was proven over time. In spatial dimension $n \geq 3$, Sylvester and Uhlmann (1987) showed, for smooth conductivities, $\Lambda_{\gamma_1} = \Lambda_{\gamma_2} \Rightarrow \gamma_1 = \gamma_2$ (using the tool of complex geometrical optics = CGO solutions). In two dimensions $n = 2$, Nachman (1996) and Astala–Päivärinta (2006) established uniqueness under weaker assumptions. That is — **the boundary DtN map determines the interior conductivity uniquely**. In principle, external measurements alone reveal the inside.

This is a remarkable result. Without cutting the body or digging the ground, from a single sheet of information — the **boundary electrical response** — the entire interior distribution is in principle fixed. The mathematical guarantee of EIT (medical imaging) and underground prospecting lies here. The inverse problem, asked correctly, is **solvable** — the cause is truly imprinted in the effect.

4. An Honest Nuance: Ill-Posedness (Instability)

But uniqueness alone is no cause for comfort. The Calderón problem is severely **ill-posed**. A **tiny measurement error** in the boundary DtN map is **greatly amplified** in the recovered interior γ . The stability estimate is **logarithmic** — raising measurement precision by orders of magnitude improves interior resolution only slightly (Alessandrini's logarithmic stability; Mandache's essential lower bound on the ill-posedness). Hence practical EIT requires **regularization** — adding prior information to stabilize the solution.

This is the honest face of inverse problems. **"Uniquely determined" and "stably recoverable" are different**. The cause is imprinted in the effect, but the imprint is **faint and easily buried in noise**. Precisely for this reason the master hones the "no-bite baseline (null)" to the utmost (Paper 89) — to read a faint deviation, the precision of the baseline is everything. The power of the inverse problem, and its instability: the discipline is to record both honestly.

5. Distinct Roots of Inverse Problems: Scattering (GLM) and Boundary-Value (Calderón)

Observation 5.1 (the same stance, different mathematics = distinct roots)

Several inverse problems now appear in the corpus. **(1) Inverse scattering** (Gel'fand–Levitan–Marchenko = GLM, Paper 89): from scattering data (reflections, phase shifts) recover the potential $V(\mathbf{x})$; the information of time and frequency is the protagonist. **(2) The inverse boundary-value problem** (Calderón, this paper): from the boundary DtN map recover the interior conductivity $\gamma(\mathbf{x})$; spatial boundary information is the protagonist. The two **share the stance of "reading the cause from the effect,"** but their equations and tools (the GLM integral equation / CGO solutions) differ — they are **different mathematics = distinct roots** (Papers 58/48). One must not mistake the similarity of stance for an identity of mathematics.

Furthermore, Mandelbrot's fractal analysis of markets (Paper 94) is inverse-problem-like in inferring hidden structure from an observed price series, but it is a **descriptive, statistical** variety — **yet another root** distinct from the deterministic inverse problems of Calderón or GLM. The corpus's "inverse problem" is one word, but beneath it stand distinct roots — **scattering, boundary-value, statistical** — the corpus's usual discipline of one word, several distinct roots.

6. Connection to the Corpus

Calderón crosses several threads of the corpus. **(a) The inverse-problem stance:** the Calderón problem is the **mathematical archetype** of Paper 89 (Zero-Tension = the inverse-problem stance). The DtN map, reading the interior from the boundary response, is the purest mathematization of "read the cause from deviations off the null." **(b) The link to Prof. Kimura:** Paper 89 recorded (at Layer 3) the author's kinship with Prof. Kimura (Kobe University) and inverse scattering. Calderón's inverse boundary-value problem is its **spatial cousin** — the same stance, distinct-root mathematics. **(c) The lineage of uniqueness:** "the boundary (little information) determines the interior (the whole)" resonates with the corpus's lineage of "little information determines structure" (the $n=2$ uniqueness of Paper 01, weak assumptions $\rightarrow$ Lie group of Paper 90) — though a different theorem, a distinct root. **(d) The author's origin:** the underground from the surface, the fish from the line, the market from the price — "read the unseen cause from the visible effect" is the core, common to Calderón's prospecting, fishing, and markets, of the author's integral brain (zoom-out and inverse problems).

The Author's Personal Note (Layer 3, non-science label / not a claim of mechanism)

In the corpus's lens (metaphor), the very act of knowing the world looks like a vast inverse problem — the Universe OS's "interior" (the true state, the hidden cause) is not directly visible, and all that reaches us is the **boundary response** (the "effects" of light, force, and data). Physics, on this view, is like EIT: reading the interior from the boundary response. But **this carries the non-science label**. "The world = a vast EIT" is a metaphor that carries intuition, not a mechanism or a prediction. **What is certain here is only the single mathematical point of the Calderón problem: the interior can be uniquely determined from boundary data (but ill-posedly, unstably)**. And a lesson not to forget — the cause is imprinted in the effect, but the imprint is faint. So the master hones the baseline (null) (89). Read the unseen from the seen, humbly, with regularization. This stance is the core of the corpus's inverse problems, and the very way of life the author drew from fishing and the markets. To Calderón, the monument of inverse problems, quiet respect is recorded.

7. Scope (What Is Shown, and What Is Not)

What is shown (descriptive, structural):

- The Calderón problem (determining the interior conductivity from the boundary DtN map) and EIT, and its uniqueness (Sylvester–Uhlmann / Nachman) (§§2–3).
- The ill-posedness of inverse problems (logarithmic stability, regularization required) = the honest nuance that unique does not mean stable (§4).
- That the corpus's inverse problems (scattering GLM, Paper 89 / boundary-value Calderón) share the stance but are distinct roots (§5).

What is not shown (restraint):

- **Do not say "unique = stable, practice is easy."** Because of ill-posedness, practice is perilous without regularization and prior information.
- **Do not identify inverse scattering, inverse boundary-value, and statistical inverse problems.** They share only the stance; the mathematics are distinct roots (58/48).
- **No new mathematics or prediction.** The Calderón problem and EIT are standard. Hence Layer 2 (structural interpretation). The OS is Layer-3 non-science.

8. Conclusion

To Paper 89's "inverse-problem stance" — reading the cause from the effect — this paper gave the **mathematical archetype** of the Calderón problem. By applying voltages at the boundary of a body and measuring currents alone, the

interior conductivity can be uniquely determined (Sylvester–Uhlmann / Nachman). The boundary determines the interior — the cause is truly imprinted in the effect. This is the foundation of EIT and the power of the inverse problem.

But honestly: the imprint is faint, and the inverse problem is severely ill-posed. Even when unique, stable recovery requires regularization. Precisely for this the master hones the baseline (null) (89). And the corpus's inverse problems — scattering (GLM) and boundary-value (Calderón) — share the stance yet are distinct roots. What this paper did is not a new claim but to give the inverse-problem stance a mathematical archetype, to separate honestly its power and instability, and to sort the corpus's inverse problems as distinct roots. Read the unseen cause from the visible effect, humbly — to this stance, common to prospecting, fishing, and markets, we return one address, together with the name of the monument, Calderón.

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