

With the Pitch Circle Fixed, Cam Contact Stress Is Least When Roller and Profile Curvature Radii Are Nearly Equal

— counting the peak Hertz line-contact stress over the rise, fixing the base circle makes a larger roller always lower the stress, while fixing the pitch circle puts the minimum inside the undercutting limit —the separator is whether the base circle or the pitch circle is held fixed — [Paper 990]

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— *From thinking to syncing.* —

Abstract

The roller size that avoids cam undercutting is ordered by deceleration, not by pressure angle (Paper 967). Neither there nor in Paper 929 was the contact stress between profile and roller treated. For cycloidal and simple harmonic rises, this paper finds the peak stress over the rise from spring and mass forces and Hertz line contact, and compares how it changes with roller radius when the base circle is fixed and when the pitch circle is fixed. No new theorem or law is claimed. First, with the base circle fixed, a larger roller lowers the stress: for a cycloidal cam with a 30 mm base circle, the stress fell from **333.861** MPa at a 4 mm roller to **139.420** MPa at 30 mm, and in all 10 cases of two curves and base circles of 5 to 30 mm it fell monotonically over rollers of 1 to 40 mm (Section 2). Second, with the pitch circle fixed, the minimum stress lies inside: for a cycloidal cam with a 30 mm pitch circle, a 5 mm roller gave **303.091** MPa, 21.5 mm gave **180.310** MPa, and 29.5 mm, just before undercutting, gave **611.151** MPa (Section 3). Third, the best roller nearly equals the profile curvature radius at the point of peak stress: dividing the best roller radius by the profile curvature radius at the point of peak stress gave **0.962550** to **1.023792** over 6 cases of two curves and three pitch circles (Section 3). Fourth, the best radius is well below the least curvature radius of the pitch curve: best radius over least pitch-curve curvature radius was **0.707727** to **0.740000** for cycloidal and **0.583333** to **0.595833** for simple harmonic (Section 3). Fifth, the separator is whether the base circle or the pitch circle is held fixed: with the base circle fixed, a larger roller also pushes the pitch curve outward, and the profile curvature radius did not fall below 26.914 mm for rollers of 4 to 30 mm (cycloidal, 30 mm base circle). With the pitch circle fixed, a larger roller cuts the profile curvature radius by its own size (Section 4). **Keywords:** cam, contact stress, Hertz contact, roller radius, pitch circle, radius of curvature.

Scope of this paper (scope note): No new theorem or law is claimed: the pressure formula for line contact between two cylinders (Hertz), that the profile becomes pointed unless the roller radius is below the pitch-curve curvature radius, and that a larger roller enlarges the equivalent contact radius, are known in contact mechanics and cam design. These are set values: a rise of 20 mm over 120 degrees, 600 rpm, follower mass 0.5 kg, spring 2000 N/m, preload 50 N, width 10 mm, steel modulus 210 GPa and Poisson ratio 0.3, base circles of 5 to 30 mm and pitch circles of 25, 30 and 40 mm are set here. They correspond to no cam. Strength is not discussed: allowable stress, life, friction, lubrication, return and dwell segments and follower offset are not treated. Only the peak stress over the rise is counted. Reasons are not separated: that the best radius comes near the profile curvature radius is only read from the form of the equivalent radius at the peak point; how that point moves with roller radius is not measured separately. Relation to earlier papers: Paper 929, in its scope note, did not design cams, leaving out cam profiles, pressure angles, contact stress and materials, and Paper 961, Paper 967, Paper 972 and Paper 977, in their scope notes, also did not treat profile strength or contact stress; this paper sits in the seat of contact stress (no earlier occupant; the pressure-angle seat was taken by Paper 961 and the undercutting seat by Paper 967). Paper 967 showed that the roller size that avoids cam undercutting is ordered by deceleration, not by pressure angle. Here the change in contact stress for roller sizes up to undercutting is counted. Paper 961 showed that cam curves with the same peak velocity need different pitch circles, set by where the pressure angle peaks. Here three pitch circles are set and the best roller is sought. What is added: finding peak stress by roller radius for two curves, confirming a monotone fall in 10 cases with the base circle fixed, confirming an interior minimum in 6 cases with the pitch circle fixed, comparing the best radius with the profile curvature radius, and placing the separator on “base circle, or pitch circle”.

1. Notation and the Question

Rise and force: a rise $h = 20$ mm over $\beta = 120$ degrees, cycloidal $s = h(u - \sin 2\pi u / (2\pi))$ or simple harmonic $s = h(1 - \cos \pi u) / 2$ ($u = \theta / \beta$). The cam turns at 600 rpm; with follower mass 0.5 kg, spring 2000 N/m and preload 50 N, the force on the follower is $F = 50 + 2000s + 0.5\ddot{s}$ and the normal force $F / \cos \phi$ (ϕ the pressure angle).

Contact stress: with $R = R_p + s$ the distance from cam centre to roller centre (R_p the pitch circle radius), the profile curvature radius is $\rho_c = \rho_p - r$ from the pitch-curve curvature radius ρ_p (r the roller radius). The equivalent radius and peak contact pressure (width 10 mm, steel) are

$$\frac{1}{R^*} = \frac{1}{r} + \frac{1}{\rho_c}, \quad p_0 = \sqrt{\frac{F_n E^*}{\pi b R^*}}, \quad E^* = \frac{E}{2(1 - \nu^2)}$$

and the maximum of p_0 over the rise is taken on a grid. With the base circle fixed, $R_p = \text{base circle} + r$; with the pitch circle fixed, R_p is set directly. The profile is taken as pointed when $\rho_p > 0$ and $\rho_c \leq 0$.

Question: how does the peak contact stress change as the roller grows, and how does that differ with what is held fixed?

2. Base Circle Fixed

Computation 2.1: base circle 30 mm.

roller	cycloidal (MPa)	angle of peak (deg)	simple harmonic (MPa)	angle of peak (deg)
4 mm	333.861	37.02	301.250	18.00
10 mm	217.516	38.40	198.639	23.58
20 mm	162.136	39.72	150.472	27.36
30 mm	139.420	40.44	130.883	29.10

For the cycloidal cam with a 30 mm roller, the least profile curvature radius was 26.914 mm. In the 10 cases of two curves and base circles of 5, 10, 15, 20 and 30 mm, varying the roller from 1 to 40 mm in 1 mm steps, peak stress fell monotonically in every case and was least at 40 mm (for cycloidal, from 203.271 MPa at a 5 mm base circle to 126.804 MPa at 30 mm).

3. Pitch Circle Fixed

Computation 3.1: best radius with the roller varied in 0.5 mm steps.

curve	pitch circle	least pitch-curve radius	best radius	stress (MPa)	stress at 5 mm	best over profile radius
cycloidal	25 mm	25.000 mm	18.5 mm	196.573	308.286	0.962550
cycloidal	30 mm	30.000 mm	21.5 mm	180.310	303.091	0.987375
cycloidal	40 mm	38.857 mm	27.5 mm	157.836	298.062	0.975829
simple harmonic	25 mm	30.000 mm	17.5 mm	187.573	276.860	0.989181
simple harmonic	30 mm	34.483 mm	20.5 mm	172.723	273.004	1.023792
simple harmonic	40 mm	43.636 mm	26.0 mm	151.816	270.869	1.005501

Just before undercutting, stress rose to 611.151 MPa at 29.5 mm for the cycloidal 30 mm pitch circle and 1109.811 MPa at 43.5 mm for the simple harmonic 40 mm pitch circle. In all 6 cases the minimum lay between the smallest radius and the radius just before undercutting. Best radius over least pitch-curve curvature radius was 0.740000, 0.716667 and 0.707727 for cycloidal and 0.583333, 0.594500 and 0.595833 for simple harmonic.

Observation 3.2

the best roller nearly equals the profile curvature radius at the point of peak stress. Best radius over the profile curvature radius at the peak point was 0.962550 to 1.023792: with the pitch circle fixed, $r + \rho_c = \rho_p$ at a given point does not depend on the roller, and the equivalent radius $r\rho_c/(r + \rho_c)$ is largest at $r = \rho_c$. The point of peak stress moves with roller radius (52.83 degrees at the best for the cycloidal 30 mm pitch circle), so this reading holds for a fixed peak point.

4. The Separator

Observation 4.1

the separator is whether the base circle or the pitch circle is held fixed. With the base circle fixed, the pitch curve widens outward by the roller's growth, and the profile curvature radius is little cut (least 26.914 mm even with a 30 mm roller on the cycloidal 30 mm base circle): with the pitch circle fixed, the pitch curve does not move and the profile curvature radius is cut by the roller's size, so too large a roller shrank the equivalent radius and raised the stress.

Observation 4.2

what is not said. Allowable stress, life, friction and lubrication, return and dwell segments, offset, trade-off with pressure-angle limits and values of real cams are not treated. What is shown is how, for the set forces and sizes, peak contact stress changes with roller radius and turns the other way depending on what is fixed.

5. Roll Call and Closing

1. Hertz line contact and rise forces (Section 1): peak over the rise.
2. base circle fixed: monotone fall (Computation 2.1): from **333.861** MPa at 4 mm.

3. pitch circle 30 mm: best 21.5 mm at **180.310 MPa** (Computation 3.1): **an interior minimum.**
4. best over profile radius **0.962550 to 1.023792** (Computation 3.1): **nearly equal.**
5. best over least pitch-curve radius **0.583333 to 0.740000** (Computation 3.1): **well inside undercutting.**
6. base circle, or pitch circle (Observation 4.1): **the separator; the scope notes of Paper 929 and others.**

Closing: With the pitch circle fixed, cam contact stress is least when roller and profile curvature radii are nearly equal. Counting peak Hertz line-contact pressure for the set rise and forces, the cycloidal cam with a 30 mm pitch circle was least at a 21.5 mm roller with 180.310 MPa, and best radius over the profile curvature radius at the peak point was 0.962550 to 1.023792 in 6 cases: with the base circle fixed, a larger roller always lowered the stress, from 333.861 MPa at 4 mm to 139.420 MPa at 30 mm. The separator is whether the base circle or the pitch circle is held fixed. Placed among the earlier papers: to the rollers and pitch circles Paper 961 set by pressure angle and Paper 967 set by undercutting, the contact stress Paper 929 and others did not treat was added. To be honest: only the peak stress over the rise was counted for a cam with set values. Whether the material holds is not said. How stress changes with roller size, and no further.

References

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