

Lapses Push the Level an Up-Down Method Tracks Upward, and Too Many Strip Strict Rules of Their Convergence Point

— with a ceiling on the proportion correct, the target of the rule lowering after k correct in a row moves up, and vanishes once the lapse rate exceeds one minus the k -th root of 0.5 —the separator is whether the ceiling on proportion correct lies above or below the target proportion — [Paper 991]

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— *From thinking to syncing.* —

Abstract

Guessing shifts the level an up-down method tracks downward, and a one-down rule has no convergence point in two-choice tasks (Paper 954). Lapses, in which an observer errs even at strengths that should be seen, were not included there. Adding a lapse rate to the observer of Paper 954 to put a ceiling on the proportion correct, this paper counts the target of the rule lowering after k correct in a row and the position of simulated observers. No new theorem or law is claimed. First, lapses move the target up: with no guessing, the target of the 3-in-a-row rule rose as 0.819329, 0.877522, 0.976026, 1.184485 and **1.504435** for lapse rates 0, 0.02, 0.05, 0.10 and 0.15 (Section 2). Second, once lapses exceed one minus the k -th root of 0.5, there is no target: that limit is 0.500000, 0.292893, 0.206299, **0.159104** and **0.129449** for $k = 1$ to 5, and at a lapse rate of 0.15 only the 5-in-a-row rule lacked a target (Section 3). Third, a rule without a target keeps rising: with lapse rate 0.15, the 5-in-a-row rule stood at 4.230500 after 600 trials and **9.631750** after 2400. With two choices it also rose, from 4.136750 to 9.657250 (Section 3). Fourth, just short of the limit, the position stayed above even with a target: with lapse rate 0.15 the 4-in-a-row rule has target 2.300495, but its position kept rising as 2.580750, 2.883000 and **3.092250** after 600, 2400 and 9600 trials, and the 600-trial estimate exceeded the target by 0.160879 (Section 4). Fifth, the separator is whether the ceiling on proportion correct lies above or below the target proportion: guessing raises the floor of proportion correct and put the one-down target of 0.5 at the floor (Paper 954). Lapses lower the ceiling and put the targets of strict rules above it. Even with guessing 0.5, at lapse rate 0.15 the 5-in-a-row rule had no target (Section 5).
Keywords: up-down method, lapse rate, psychometric function, target level, guessing, psychophysics.

Scope of this paper (scope note): No new theorem or law is claimed: putting lapses into the psychometric function so that the ceiling on proportion correct falls below 1, that up-down methods track the k -th root of 0.5 (Levitt), and that lapses bias threshold estimates (Wichmann and Hill), are known in psychophysics. These are set values: the observer with guessing and lapses added to a normal cumulative curve, lapse rates 0 to 0.15, step 0.1, start 2.0, 400 runs and 600, 2400 and 9600 trials are added here to the models of Paper 935 and Paper 954, and correspond to no sense or experiment. Lapses are constant: the lapse rate is constant over trials; lapses that change with trials, like fatigue or habituation, are not included. Convergence is not claimed: for the rule just short of the limit, settling of the position within 9600 trials was not confirmed even though it has a target. Relation to earlier papers: Paper 954, in its scope note, did not treat actual perception, leaving out fatigue, habituation and lapses; this paper sits in the seat of lapses (no earlier occupant; Paper 935's scope note also left out fatigue and habituation, but this paper does not sit in the seat of what changes with trials). Paper 954 showed that guessing shifts the level an up-down method tracks downward, and that a one-down rule has no convergence point in two-choice tasks. Here, moving the ceiling rather than the floor of proportion correct, where the target moves and in which rules it vanishes are counted. Paper 988 showed that guessing spreads up-down estimates on the stimulus scale. Here not the spread but the position of the target, and whether it exists, is examined. What is added: solving targets for two guessing rates, five lapse rates and five rules, giving the lapse limit at which the target vanishes, confirming with long runs that rules without a target keep rising, showing that just short of the limit the position also kept rising, and placing the separator on "ceiling above or below the target".

1. Notation and the Question

Observer: adding a lapse rate λ to the guessing rate γ of Paper 954, the proportion correct at stimulus strength x is

$$p(x) = \gamma + (1 - \gamma - \lambda) \Phi(x)$$

so that p moves between γ (floor) and $1 - \lambda$ (ceiling).

Rule and target: after k correct in a row x is lowered by 0.1, and after any error raised by 0.1. The start is 2.0. The rule tracks proportion $0.5^{1/k}$, and the target solves $p(x) = 0.5^{1/k}$. A solution exists only when $\gamma < 0.5^{1/k} < 1 - \lambda$, that is, for lapses, $\lambda < 1 - 0.5^{1/k}$. Simulated observers were run 400 times for 600 trials (the estimate being the mean position over the last two thirds), and three combinations were extended to 2400 and 9600 trials.

Question: when lapses put a ceiling on proportion correct, where does a rule's target move, and in which rules does it vanish?

2. Lapses and Targets

Computation 2.1: targets with no guessing (none means no solution).

lapse rate	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
0	−0.000000	0.544952	0.819329	0.998149	1.128998
0.02	0.025581	0.587415	0.877522	1.071633	1.217626
0.05	0.066012	0.656731	0.976026	1.201153	1.381055
0.10	0.139710	0.791501	1.184485	1.508834	1.842219
0.15	0.223008	0.961662	1.504435	2.300495	none

For each of $k = 1$ to 4, the target rose with the lapse rate. The values at lapse rate 0 are those of Paper 935 and Paper 954.

Computation 2.2: 600-trial estimate minus target (no guessing).

lapse rate	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
0	0.005087	0.011682	0.019513	0.009253	0.026901
0.10	0.003447	0.027381	0.037874	0.097214	0.126704
0.15	0.006577	0.042548	0.105255	0.160879	none

Observation 2.3

the larger the lapse rate and the stricter the rule, the further the estimate stayed above the target. At lapse rate 0 the difference was at most 0.026901, but at 0.15 with the 4-in-a-row rule it was 0.160879: for $k = 1$ the difference stayed at most 0.006577 whatever the lapse rate.

3. The Limit Where the Target Vanishes

Computation 3.1: the lapse limit $1 - 0.5^{1/k}$ at which the target vanishes is 0.500000, 0.292893, 0.206299, 0.159104 and 0.129449 for $k = 1$ to 5. A lapse rate of 0.15 lies above the $k = 5$ limit and below the $k = 4$ limit, and with no guessing only $k = 5$ lacked a target.

Computation 3.2: positions of combinations without a target (mean over 400 runs).

combination	after 600 trials	after 2400 trials
guessing 0, lapse 0.15, $k = 5$	4.230500	9.631750
guessing 0.5, lapse 0.15, $k = 5$	4.136750	9.657250

Both stood more than 1 higher after 2400 trials than after 600. With guessing 0.5, $k = 1$ had no target whatever the lapse rate and moved down (-2.861000 after 600 trials at lapse rate 0.15).

4. Just Short of the Limit

Computation 4.1: with no guessing, lapse rate 0.15 and $k = 4$ (target 2.300495), the position was 2.580750 after 600 trials, 2.883000 after 2400 and 3.092250 after 9600 (a difference of 0.791755 from the target), and the mean over the last two thirds of 9600 trials was 3.039208.

Observation 4.2

even with a target, just short of the limit the position stayed above it. Settling of the position within 9600 trials could not be confirmed: the ceiling $1 - 0.15$ and the target proportion 0.840896 are close. Why the position stays above was not measured term by term, and runs longer than 9600 trials were not counted.

5. The Separator

Observation 5.1

the separator is whether the ceiling on proportion correct lies above or below the target proportion. Guessing raises the floor, and in two-choice tasks put the one-down target of 0.5 at the floor, sending the rule downward (Paper 954): lapses lower the ceiling and put the targets of strict rules above it, sending them upward. Even with guessing 0.5, the 5-in-a-row rule at lapse rate 0.15 had no target; floor and ceiling select rules separately.

Observation 5.2

what is not said. Lapses that change with trials (fatigue or habituation), estimating the lapse rate, which rule to use, and real perception experiments are not treated. What is shown is how, when lapses lower the ceiling, the target of an up-down method moves up, and in which rules and when it vanishes.

6. Roll Call and Closing

1. an observer with guessing and lapses (Section 1): **floor and ceiling**.
2. 3-in-a-row target **0.819329** to **1.504435** (Computation 2.1): **moves up**.
3. limits **0.159104** and **0.129449** ($k = 4, 5$) (Computation 3.1): **beyond them no target**.
4. a rule without a target at **9.631750** after **2400** trials (Computation 3.2): **keeps rising**.
5. $k = 4$ just short of the limit at **3.092250** after **9600** trials (Computation 4.1): **stays above**.
6. ceiling above or below the target (Observation 5.1): **the separator; the scope note of Paper 954**.

Closing: Lapses push the level an up-down method tracks upward, and too many strip strict rules of their convergence point. Adding lapses to the observer of Paper 954, the 3-in-a-row target rose from 0.819329 at lapse rate 0 to 1.504435 at 0.15, and targets vanished once lapses exceeded $1 - 0.5^{1/k}$ (0.159104 for $k = 4$, 0.129449 for $k = 5$): the 5-in-a-row rule without a target rose to 9.631750 after 2400 trials, and the 4-in-a-row rule just short of the limit also stayed above its target at 3.092250 after 9600 trials. The separator is whether the ceiling on proportion correct lies above or below the target proportion. Placed among the earlier papers: to the up-down methods Paper 954 counted by moving the floor with guessing, the other side, moving the ceiling with the lapses Paper 954 did not include, was added. To be honest. This counts targets and positions for one observer with a constant lapse rate. How a ceiling moves and removes a rule's target, and no further.

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