

Massive Fragments Lower the Share Extracted in One Split, and Mass on the Infalling Fragment Weighs More

— solving the equatorial Kerr split from conservation and mass-shell conditions, equal-mass fragments give a share matching the photon formula with $2/r$ multiplied by $1 - 4\mu^2$, and nothing is extracted once μ exceeds half the root of $1 - r/2$ —the separator is whether the mass is given to the escaping fragment or to the one falling into the hole — [Paper 995]

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September 2026

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— *From thinking to syncing.* —

Abstract

The formula for the gain in one split does not ask whether the falling particle can get there (Paper 939). There the fragments were limited to two photons, and massive fragments were not treated. For the same equatorial Kerr split as Paper 939, this paper finds, by solving conservation and mass-shell conditions numerically, the share by which the escaping fragment's energy exceeds the incoming energy when both fragments have rest mass. No new theorem or law is claimed. First, the heavier the fragments, the smaller the share extracted: for spin $a = 1$ just outside the horizon ($r = 1.0001$), equal-mass fragments of mass 0, 0.1, 0.2 and 0.3 gave **0.207071**, 0.192786, 0.148042 and 0.065657, and mass 0.4 gave nothing (Section 2). Second, for equal masses the share matched the photon formula with one factor: at the 18 points where something was extracted, the share agreed with $(\sqrt{(2/r)(1 - 4\mu^2)} - 1)/2$ within 2.171×10^{-8} , the same form holding for spin 0.9. The formula was not derived (Section 2). Third, fragment mass has an upper limit: the largest mass allowing extraction, found by bisection, was **0.353536**, 0.316228, 0.250000 and 0.158114 at $r = 1.0001, 1.2, 1.5$ and 1.8 , agreeing with half the root of $1 - r/2$ (Section 3). Fourth, mass on the infalling fragment weighs more: at $r = 1.0001$, giving mass 0.3 only to the escaping fragment gave 0.188435, and only to the infalling fragment **0.098435** (Section 4). Fifth, the separator is whether the mass is given to the escaping fragment or to the one falling into the hole: with total mass 0.4, 0.3 out and 0.1 in gave 0.174949, and 0.1 out and 0.3 in gave 0.094949, neither matching simple forms using only the sum or the sum of squares of the masses (Section 4). **Keywords:** Penrose process, Kerr black hole, ergosphere, fragment mass, energy extraction, general relativity.

Scope of this paper (scope note): No new theorem or law is claimed: the Penrose process, the ergosphere of a Kerr black hole, and that massive fragments extract a smaller share than photons, are known in general relativity. Units are set: the gravitational constant, the speed of light and the hole's mass are 1, and the splitting particle's mass is also 1. The split is limited: only a particle at rest far away splitting at its turning point on the equatorial plane, with both fragments also turning there, is treated. Splits off the equatorial plane, multi-stage splits and other choices of fragment directions are not. Reachability is not asked: whether the falling particle can reach the splitting point (Paper 939) is not checked; only local conditions are counted. Formulas are not derived: the forms for equal-mass shares and the mass limit were only checked against numerical solutions, not derived. Relation to earlier papers: Paper 939, in its scope note, limited the split, leaving out thrown particles, massive fragments, splits off the equatorial plane and multi-stage splits; this paper sits in the seat of massive fragments (no earlier occupant). Paper 939 showed that the formula for the gain in one split does not ask whether the falling particle can get there. Here, under the same local conditions, the fragments are given mass. Paper 887 showed that the total energy extractable from a rotating black hole is at most 0.292893 of its mass. Here not the total but the share in one split is counted. What is added: finding shares numerically for two spins, three radii and seven masses, confirming that equal-mass shares match one form, finding the upper limit of mass by bisection, comparing mass on the escaping and on the infalling fragment, and placing the separator on "which fragment carries the mass".

1. Notation and the Question

Split: with the gravitational constant, the speed of light and the hole's mass set to 1, on the equatorial plane of a Kerr spacetime with spin a , a particle of mass 1 at rest far away (energy $E_0 = 1$) splits at its turning point at radius r . Fragment i has rest mass μ_i , energy E_i and angular momentum L_i , and also turns there. The turning (mass-shell) condition is

$$[E(r^2 + a^2) - aL]^2 = \Delta[\mu^2 r^2 + (L - aE)^2], \quad \Delta = r^2 - 2r + a^2$$

and solutions with $E_1 + E_2 = E_0$, $L_1 + L_2 = L_0$, both fragments future-directed and the infalling fragment's energy E_2 negative were sought, taking the largest share $\eta = (E_1 - E_0)/E_0$. L_2 was stepped over a wide range and sign changes refined by bisection.

Question: when fragments have rest mass, how does the share extracted in one split fall, and how does it depend on which fragment carries the mass?

2. Equal-Mass Fragments

Computation 2.1: share extracted (none means no negative-energy solution).

spin	radius	mass 0	0.1	0.2	0.3	0.4
1	1.000100	0.207071	0.192786	0.148042	0.065657	none
1	1.200000	0.145497	0.132456	0.091608	0.016398	none
1	1.800000	0.027046	0.016398	none	none	none
0.9	1.435990	0.090078	0.078156	0.040815	none	none
0.9	1.500000	0.077350	0.065685	0.029150	none	none
0.9	1.800000	0.027046	0.016398	none	none	none

Masses 0.45 and 0.49 gave nothing at any point. Mass 0 matched the photon closed form $(\sqrt{2/r} - 1)/2$.

Computation 2.2: at the 18 points where something was extracted, the largest difference between the share and

$$\eta = \frac{1}{2} \left(\sqrt{\frac{2}{r} (1 - 4\mu^2)} - 1 \right)$$

was 2.171×10^{-8} . The points with nothing extracted coincided with those where this form is 0 or below. Spin 1 and spin 0.9 at $r = 1.8$ gave the same values; spin does not enter this form. That $r = 1.2$ with mass 0.3 and $r = 1.8$ with mass 0.1 both gave 0.016398 is because $(2/r)(1 - 4\mu^2)$ takes the same value in this form.

3. Upper Limit of Mass

Computation 3.1: for spin 1, the largest equal mass allowing extraction, found by bisection, was 0.353536, 0.316228, 0.250000 and 0.158114 at $r = 1.0001, 1.2, 1.5$ and 1.8 , agreeing with $\frac{1}{2} \sqrt{1 - r/2}$ within 10^{-4} .

Observation 3.2

the deeper the split, the heavier the fragments that still extract energy. The limit was 0.353536 just outside the horizon and 0.158114 at $r = 1.8$ near the edge of the ergosphere: every limit was below 0.5, where the sum of the two fragment masses would equal the splitting particle's mass 1.

4. Which Fragment Carries the Mass

Computation 4.1: spin 1, $r = 1.0001$, with unequal fragment masses.

escaping fragment	infalling fragment	share extracted	sum form	sum-of-squares form
0.3	0	0.188435	0.174503	0.140280
0	0.3	0.098435	0.174503	0.140280
0.3	0.1	0.174949	0.148042	0.132424
0.1	0.3	0.094949	0.148042	0.132424

The sum form replaces $1 - 4\mu^2$ with $1 - (\mu_1 + \mu_2)^2$, and the sum-of-squares form with $1 - 2(\mu_1^2 + \mu_2^2)$; neither matched unequal fragments.

Observation 4.2

mass on the infalling fragment weighs more. The same mass 0.3 gave 0.188435 on the escaping fragment and 0.098435 on the infalling one: swapping the pair of masses left the sum and the sum of squares unchanged, but changed the share.

5. The Separator

Observation 5.1

the separator is whether the mass is given to the escaping fragment or to the one falling into the hole. For equal masses the share matched one form, the photon formula with $2/r$ multiplied by $1 - 4\mu^2$: with unequal masses, neither simple replacement expressed it, and the share fell more the more mass was on the infalling side. A form for unequal fragments was not sought.

Observation 5.2

what is not said. Whether the falling particle reaches the splitting point, splits off the equatorial plane, multi-stage splits, derivations of the forms, and real astronomical objects or jets are not treated. What is shown is how fragment mass lowers the share extracted in one split solved under local conditions.

6. Roll Call and Closing

1. a split solved from conservation and mass shells (Section 1): **both fragments also turn.**
2. **0.065657** at mass **0.3**, nothing at **0.4** (Computation 2.1): **down from the photon 0.207071.**
3. difference **2.171×10^{-8}** from the equal-mass form (Computation 2.2): **spin does not enter.**
4. mass limit **0.353536** to **0.158114** (Computation 3.1): **half the root of 1 minus $r/2$.**
5. **0.098435** with mass on the infalling fragment (Computation 4.1): **0.188435** on the escaping one.
6. escaping, or infalling (Observation 5.1): **the separator; the scope note of Paper 939.**

Closing: Massive fragments lower the share extracted in one split, and mass on the infalling fragment weighs more. Solving the split of Paper 939 from conservation and mass-shell conditions, just outside the horizon at spin 1 the equal-mass share fell from 0.207071 at mass 0 to 0.065657 at 0.3, and nothing was extracted at 0.4: equal-mass shares agreed with $(\sqrt{(2/r)(1 - 4\mu^2)} - 1)/2$ within 2.171×10^{-8} , and the mass limit with half the root of $1 - r/2$. Mass 0.3 on the infalling fragment gave 0.098435, and on the escaping fragment 0.188435. The separator is whether the mass is given to the escaping fragment or to the one falling into the hole. Placed among the earlier papers: to the one split Paper 939 limited to two photons, the fragment mass Paper 939 did not treat was added. To be honest: numbers for splits solved under local conditions were set out and checked against an equal-mass form. The form was not derived. How fragment mass lowers the share extracted, and no further.

References

1. Bardeen, J. M., Press, W. H., & Teukolsky, S. A. (1972). Rotating black holes: locally nonrotating frames, energy extraction, and scalar synchrotron radiation. *The Astrophysical Journal*.
2. Yamagishi, Y. (2026). *Extractable Energy from a Rotating Black Hole Starts to Grow as the Square of the Spin and Reaches at Most 0.292893 of the Mass* [Paper 887]. YOI: <https://www.yamagishiyuuki.com/yoi/887>
3. Yamagishi, Y. (2026). *The Formula for the Gain in One Split Does Not Ask Whether the Falling Particle Can Get There* [Paper 939]. YOI: <https://www.yamagishiyuuki.com/yoi/939>

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