

With Ground Contact Time, Jump Rope Needs Less Height, and a Double Under Needs More Than Four Times a Single

— the rope keeps turning while the feet are on the ground, so the time in the air is the rope's turning time minus the contact time, and no jump is possible once contact lasts longer than one turn of the rope —the separator is whether the contact time is shorter or longer than one turn of the rope — [Paper 996]

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Abstract

A double under needs four times the height of a single, and turning the rope twice as fast brings it back to the same height (Paper 881). The time the feet stay on the ground between landing and take-off was not included there. Adding ground contact time to the jump rope of Paper 881, this paper finds the heights needed for single, double and triple jumps and the condition for being able to jump at all. No new theorem or law is claimed. First, with contact time, the height needed falls: turning 2 times per second, contact times of 0, 0.1, 0.2 and 0.3 s gave single-jump heights of 0.306563, 0.196200, 0.110363 and **0.049050** m, and double-jump heights of 1.226250, 0.993263, 0.784800 and 0.600862 m (Section 2). Second, the double-to-single height ratio exceeds four: at 2 turns per second the ratio rose as 4.000000, 5.062500, **7.111111** and 12.250000, and in every combination it agreed with the square of $(2 \text{ minus turning rate times contact time})$ over $(1 \text{ minus turning rate times contact time})$ (Section 2). Third, no jump is possible when contact lasts longer than one turn of the rope: at 4 turns per second (0.2500 s per turn) with contact 0.3 s, and at 6 turns per second (0.1667 s) with 0.2 and 0.3 s, neither single nor double jumps were possible (Section 3). Fourth, twice the turning rate for the same height survives for equal contact time: a double at 4 turns per second and a single at 2 turns per second both needed **0.248316** m with contact 0.05 s, agreeing in every combination compared (Section 4). Fifth, the separator is whether the contact time is shorter or longer than one turn of the rope, while shorter, contact removes the same time from the air, lowering heights and pushing the ratio away from four. Near one turn, the single-jump height approaches 0 (0.001362 m at 3 turns per second and 0.3 s) and the ratio rose to **121.000000** (Section 5). **Keywords:** jump rope, double under, ground contact time, time in the air, jump height, projectile motion.

Scope of this paper (scope note): No new theorem or law is claimed: the relation between jump height and time in the air, the relation between rope turns per jump and time in the air, and that a jumper spends time on the ground, are known from projectile mechanics and observation of rope jumping. These are set values: turning rates of 2, 3, 4 and 6 per second and the gravitational acceleration are those of Paper 881, and contact times of 0 to 0.3 s are set here. They correspond to no athlete's record. Contact time is constant: contact time does not depend on the style or height of the jump, the rope turns at constant speed, and it suffices for the rope's passes under the feet to fall within the time in the air. Rope slack, air resistance and arm motion are not included. No training advice: how to jump is not stated. Relation to earlier papers: Paper 881, in its scope note, did not include time with the feet on the ground, leaving out the time between landing and take-off, rope slack and air resistance; this paper sits in the seat of ground contact time (no earlier occupant; Paper 947 treated the rope-shape side of Paper 881 and also left this time out). Paper 881 showed that a double under needs four times the height of a single, and that turning the rope twice as fast brings it back to the same height. Here the change in that fourfold ratio and same height when contact time is added is counted. What is added: finding single, double and triple heights for four turning rates and six contact times, confirming that the height ratio agrees with one formula, dividing impossible combinations by contact time and one turn of the rope, confirming that twice the rate still gives the same height, and placing the separator on "contact time against one turn of the rope".

1. Notation and the Question

Setting: as in Paper 881, the rope turns ω times per second, passes under the feet k times per jump, and the gravitational acceleration is $g = 9.81 \text{ m/s}^2$. Let t_c be the time the feet are on the ground. The rope keeps turning during contact, so of the jump cycle k/ω the time in the air and the height are

$$t = \frac{k}{\omega} - t_c, \quad h = \frac{gt^2}{8}$$

The k passes are $1/\omega$ apart, so fitting all of them into the air needs $t \geq (k - 1)/\omega$, that is $t_c \leq 1/\omega$. Counts were made for $\omega = 2, 3, 4, 6$, $t_c = 0, 0.05, 0.1, 0.15, 0.2, 0.3 \text{ s}$ and $k = 1, 2, 3$.

Question: with ground contact time, how do the height needed for a double under and its ratio to a single change?

2. Heights and Ratios

Computation 2.1: 2 turns per second (0.5000 s per turn).

contact time	single (m)	double (m)	triple (m)	double over single
0 s	0.306563	1.226250	2.759062	4.000000
0.05 s	0.248316	1.106691	2.578191	4.456790
0.1 s	0.196200	0.993263	2.403450	5.062500
0.15 s	0.150216	0.885966	2.234841	5.897959
0.2 s	0.110363	0.784800	2.072363	7.111111
0.3 s	0.049050	0.600862	1.765800	12.250000

The values at contact time 0 are those of Paper 881. In every combination of turning rate and contact time, double over single agreed with $((2 - \omega t_c)/(1 - \omega t_c))^2$ within 10^{-9} .

Observation 2.2

the longer the contact, the lower the heights and the higher the ratio.Contact shortens the time in the air by its own length: the single’s time in the air is $1/\omega - t_c$ and the double’s $2/\omega - t_c$; the same t_c is subtracted, so the single shrinks more as a proportion.

3. Impossible Combinations

Computation 3.1: other turning rates (none means no jump possible).

per second	one turn (s)	contact time	single (m)	double (m)	double over single
3 turns	0.3333	0.2 s	0.021800	0.267050	12.250000
3 turns	0.3333	0.3 s	0.001362	0.164862	121.000000
4 turns	0.2500	0.2 s	0.003066	0.110363	36.000000
4 turns	0.2500	0.3 s	none	none	none
6 turns	0.1667	0.15 s	0.000341	0.041216	121.000000
6 turns	0.1667	0.2 s	none	none	none

Every combination with contact longer than one turn of the rope allowed no single, double or triple jump.

4. Same Height at Twice the Rate

Computation 4.1: a double at 2ω turns per second has time in the air $2/(2\omega) - t_c = 1/\omega - t_c$, the same as a single at ω . A double at 4 and a single at 2 turns per second both gave 0.248316 m with contact 0.05 s and 0.196200 m with 0.1 s, and a double at 6 and a single at 3 both gave 0.098441 m with 0.05 s, agreeing within 10^{-12} in every combination compared.

Observation 4.2

the same height survives, but the allowed contact time shortens. A single at 2 turns per second was possible even with contact 0.3 s (0.049050 m): the double at 4 turns per second, which should need the same height, was impossible at 0.3 s, longer than its 0.2500 s per turn.

5. The Separator

Observation 5.1

the separator is whether the contact time is shorter or longer than one turn of the rope. While shorter, contact removed the same time from the air, lowering heights and pushing the ratio away from four: near one turn, the single's time in the air approached 0, giving 0.001362 m and a ratio of 121.000000 at 3 turns per second and 0.3 s. Beyond one turn, the rope passes under the feet while they are on the ground, and no jump is possible.

Observation 5.2

what is not said. Contact time that changes with style or height, rope slack, air resistance, arm motion, how to jump, and athletes' values are not treated. What is shown is how a constant contact time changes the height needed for a double under and its ratio to a single, and when jumping becomes impossible.

6. Roll Call and Closing

1. time in the air = turning time minus contact time (Section 1): the condition for passes in the air.

2. single **0.049050 m** at **0.3 s** (Computation 2.1): **heights fall**.
3. double over single **7.111111 (0.2 s)** (Computation 2.1): **more than four**.
4. impossible beyond one turn (Computation 3.1): **0.3 s at 4 turns per second**.
5. a double at twice the rate needs the same **0.248316 m** (Computation 4.1): **survives for equal contact time**.
6. contact time against one turn of the rope (Observation 5.1): **the separator; the scope note of Paper 881**.

Closing: With ground contact time, jump rope needs less height, and a double under needs more than four times a single. Adding contact time to the jump rope of Paper 881, at 2 turns per second the single fell to 0.049050 m with 0.3 s, and double over single reached 7.111111 with 0.2 s: combinations with contact longer than one turn were impossible, and a double at twice the rate needed the same height as a single for equal contact time (0.248316 m at 0.05 s). The separator is whether the contact time is shorter or longer than one turn of the rope. Placed among the earlier papers: to the jump rope Paper 881 counted by time in the air alone, the ground contact time Paper 881 did not include was added. To be honest: heights were counted with one formula and a constant contact time. How contact time changes heights and ratios, and when jumping becomes impossible, and no further.

References

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