

In a Finite Forest the Best Number of Trees per Plot Holds, Until the Budget Covers Every Plot and More Trees Become Best

— sampling plots and trees without replacement, the finite-population correction lowers the variance by a constant, the plot-mean variance over the number of plots, leaving the best number per plot independent of budget, until the number of plots reaches the forest's plots and the best moves to more trees —the separator is whether the budget sets the number of plots or the forest's plot count caps it — [Paper 998]

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Abstract

The best number of trees to measure per plot ignores the budget, and doubling or halving it costs the same (Paper 980). The population was taken as infinite there, without finite-population correction. Sampling plots and trees without replacement from the same forest of 400 plots with 50 trees each as Paper 980, this paper finds by exhaustive search the number of plots and trees per plot minimising the corrected variance under a budget, and compares with the formula for an infinite population. No new theorem or law is claimed. First, while the budget cannot cover every plot, the best number per plot does not change: with costs of 20 per plot and 1 per tree, budgets of 1000, 2000, 5000, 10000 and 14000 all gave a best of 17 trees per plot (27, 54, 135, 270 and 378 plots), the same as the infinite-population formula (Section 2). Second, the correction lowers the variance by a constant: for those five budgets, the infinite-population variance minus the corrected variance was always the plot-mean variance over the number of plots, **2.943591** (Section 2). Third, once the budget covers every plot, more trees become best: a budget of 20000 made 400 plots with **30** trees each best (variance 0.420318), and 28000 made 400 plots with 50 trees best (variance 0). The infinite-population formula chose 540 and 756 plots, numbers the forest does not have (Section 3). Fourth, random sampling without replacement matches the corrected formula: the variance over 4000 draws was 1.020748, **1.000904** and 0.984477 times the formula for 55 plots with 16 trees, 400 with 30 and 200 with 50 (Section 4). Fifth, the separator is whether the budget sets the number of plots or the forest's plot count caps it: the corrected formula is the infinite one minus an amount depending on neither the number of plots nor trees per plot, so while not every plot could be measured the best stayed at 17 trees. With a cap on plots, the remaining budget went to trees (Section 5). **Keywords:** two-stage sampling, finite-population correction, optimal subsample size, survey sampling, survey cost, forest inventory.

Scope of this paper (scope note): No new theorem or law is claimed: the variance formula for two-stage sampling without replacement from a finite population, and that the best-number formula keeps the same form with finite-population correction, are known in sampling theory. These are set values: the forest is the same random forest of 400 plots with 50 trees as Paper 980, and costs of 20 per plot and 1 per tree, budgets of 1000 to 28000 and 4000 random draws are set here. They correspond to no forest. Sampling is simplified: plots are equal in size with equal trees per plot; combination with strata, varying trees per plot and travel costs between plots are not included. Any leftover budget is left unused. No measuring plan is recommended: how many trees a real survey should measure is not treated. Relation to earlier papers: Paper 980, in its scope note, simplified sampling, taking the population as infinite and leaving out finite-population correction, combination with strata, travel costs between plots and varying the number per plot; this paper sits in the seat of finite-population correction (no earlier occupant). Paper 980 showed that the best number of trees to measure per plot ignores the budget, and that doubling or halving it costs the same. Here, in a finite forest, how far that best number holds and where it moves is counted. What is added: finding the corrected and infinite-population bests by exhaustive search for seven budgets, confirming that the correction differs by a constant, showing that the best moves to more trees once the budget covers every plot, checking the formula by sampling without replacement, and placing the separator on “the budget, or the cap on plots”.

1. Notation and the Question

Forest and formula: as in Paper 980, a forest of $N = 400$ plots with $M = 50$ trees each is made (log-normal with plot-effect share 0.1). With S_1^2 the variance of plot means and S_2^2 the mean within-plot variance, the variance of the mean of n plots and m trees per plot drawn without replacement is

$$V(n, m) = \left(1 - \frac{n}{N}\right) \frac{S_1^2}{n} + \left(1 - \frac{m}{M}\right) \frac{S_2^2}{nm}$$

and the infinite-population formula is $(S_1^2 - S_2^2/M)/n + S_2^2/(nm)$. For this forest $S_1^2 = 1177.436451$, $S_2^2 = 12609.537335$ and $S_1^2 - S_2^2/M = 925.245704$, and the best-number formula for costs of 20 per plot and 1 per tree, $m^* = \sqrt{20S_2^2/(S_1^2 - S_2^2/M)}$, gave 16.5096. Under budget B , $n = \lfloor B/(20 + m) \rfloor$ (not above 400 when corrected), and $m = 1$ to 50 was searched exhaustively.

Question: when plots and trees are drawn without replacement from a finite forest, how do the best numbers of plots and trees change?

2. Budgets That Cannot Cover Every Plot

Computation 2.1: best combinations and variances.

budget	corrected (plots, trees)	variance	infinite (plots, trees)	variance
1000	27, 17	58.796527	27, 17	61.740118
2000	54, 17	27.926468	54, 17	30.870059
5000	135, 17	9.404433	135, 17	12.348024
10000	270, 17	3.230421	270, 17	6.174012
14000	378, 17	1.466417	378, 17	4.410008

For the five budgets, infinite minus corrected variance was 2.943591, agreeing with $S_1^2/N = 2.943591$ within 10^{-9} .

Observation 2.2

the correction lowers the variance by a constant and does not move the best number. Expanding the corrected formula gives the infinite formula minus S_1^2/N : the subtracted S_1^2/N depends on neither n nor m , so the best combination was the same under both formulas.

3. Budgets That Cover Every Plot

Computation 3.1: budgets 20000 and 28000.

budget	corrected (plots, trees)	variance	infinite (plots, trees)	variance
20000	400, 30	0.420318	540, 17	3.087006
28000	400, 50	0.000000	756, 17	2.205004

Observation 3.2

with a cap on plots, the best moves toward more trees. Keeping 17 trees at a budget of 20000 would need 540 plots, but the forest has only 400: stopping at 400 plots, putting the remaining budget into trees made 30 trees best. At 28000 all 50 trees in 400 plots could be measured, and the variance was 0. The infinite formula made numbers of plots the forest does not have best.

4. Checking by Random Sampling

Computation 4.1: variance over 4000 draws of plots and trees without replacement.

plots	trees per plot	sampled variance	corrected formula	ratio
55	16	28.793328	28.208078	1.020748
400	30	0.420698	0.420318	1.000904
200	50	2.897897	2.943591	0.984477

In all three, the sampled variance was between 0.95 and 1.05 times the formula. For 200 plots with 50 trees, the formula equals $S_1^2/400$, 2.943591.

5. The Separator

Observation 5.1

the separator is whether the budget sets the number of plots or the forest’s plot count caps it. While the budget sets the number of plots, the correction only subtracts a constant, and the best number stayed at 17, near 16.5096: once the forest’s plot count caps the number of plots, more plots cannot be added, the remaining budget is best spent on trees, and a budget covering every tree brings the variance to 0.

Observation 5.2

what is not said. Combination with strata, varying trees per plot, travel costs, allocations that use up the budget, and how many trees a real survey should measure are not treated. What is shown is when the best number is kept and when it moves in sampling without replacement from a finite forest.

6. Roll Call and Closing

1. two-stage sampling without replacement (Section 1): the forest of Paper 980.
2. best 17 trees up to budget 14000 (Computation 2.1): unmoved by the correction.
3. correction difference **2.943591** (Computation 2.1): constant, plot-mean variance over plots.

4. **400 plots with 30 trees at 20000** (Computation 3.1): **moves toward trees.**
5. **sampling at 1.000904 of the formula and others** (Computation 4.1): **matches the formula.**
6. **the budget, or the cap on plots** (Observation 5.1): **the separator; the scope note of Paper 980.**

Closing: In a finite forest the best number of trees per plot holds, until the budget covers every plot and more trees become best. Drawing plots and trees without replacement from the forest of Paper 980, budgets of 1000 to 14000 kept a best of 17 trees per plot, and the correction lowered the variance by a constant 2.943591: a budget of 20000 made 400 plots with 30 trees best, 28000 made all trees best, and the infinite formula chose 540 or 756 plots, numbers the forest lacks. Sampling without replacement gave 1.020748 to 0.984477 times the formula. The separator is whether the budget sets the number of plots or the forest's plot count caps it. Placed among the earlier papers: to the best number Paper 980 counted for an infinite population, the finite-population correction Paper 980 did not include was added. To be honest: in one random forest, the corrected formula was searched exhaustively and checked by sampling. When the best number holds and when it moves, and no further.

References

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