

Splitting a Particle Thrown in Faster Extracts No More Energy, and the Harder It Is Thrown, the Less

— raising the energy at infinity E_0 above 1 in the equatorial Kerr split, the extracted amount matches one half of the root of $(E_0^2 - 1 + 2/r) - E_0$ and falls as E_0 grows, increasing only when the split point is made deeper — the separator is whether energy is added far away or the split point is made deeper — [Paper 999]

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September 2026

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Abstract

The formula for the gain in one split does not ask whether the falling particle can get there (Paper 939). Only a particle at rest far away was treated there, and particles thrown in with speed were not. For the same equatorial Kerr split as Paper 939, this paper finds, by solving conservation and photon mass-shell conditions numerically, the amount and share extracted by the escaping photon when a thrown particle (energy at infinity E_0 above 1) splits into two photons. No new theorem or law is claimed. First, the harder the throw, the less extracted: for spin 1 just outside the horizon ($r = 1.0001$), $E_0 = 1, 1.1, 1.5, 2, 5$ and 10 gave extracted amounts of **0.207071**, **0.193270**, **0.151360**, **0.118012**, **0.049500** and **0.024933** (Section 2). Second, the extracted share falls faster still: at the same point the share $(E_1 - E_0) / E_0$ fell from **0.207071** at $E_0 = 1$ to **0.059006** at 2 and **0.002493** at 10 (Section 2). Third, the extracted amount matched one form: at 24 points of four radii and six energies, the amount agreed with one half of $(\sqrt{E_0^2 - 1 + 2/r} - E_0)$ within **9.030×10^{-8}** . The form was not derived (Section 3). Fourth, thrown hard, the extracted amount shrinks in inverse proportion to the energy thrown in: in this form E_0 times the extracted amount was **0.249950** at $r = 1.0001$ with $E_0 = 1000$ and **0.083333** at $r = 1.5$, agreeing with $(2/r - 1)/4$ (Section 3). Fifth, the separator is whether energy is added far away or the split point is made deeper: for every E_0 , moving the split point from $r = 1.8$ to just outside the horizon increased the amount extracted (from **0.013794** to **0.118012** at $E_0 = 2$). Adding energy far away decreased it at every radius (Section 4). **Keywords:** Penrose process, Kerr black hole, ergosphere, thrown particle, energy extraction, general relativity.

Scope of this paper (scope note): No new theorem or law is claimed: the Penrose process, the ergosphere of a Kerr black hole, and that the amount extractable changes with the speed of the splitting particle, are known in general relativity. Units are set: the gravitational constant, the speed of light and the hole's mass are 1, the spin is 1, and the thrown particle's mass is 1. The split is limited: only a thrown particle splitting into two photons at its turning point on the equatorial plane, with the photons also turning there, is treated. Extraction by collisions of two particles, combination with massive fragments, splits off the equatorial plane and multi-stage splits are not. Reachability is not asked: whether the thrown particle reaches the split point (Paper 939) is not checked; only local conditions are counted. The form is not derived: the form of the extracted amount was only checked against numerical solutions, not derived. Relation to earlier papers: Paper 939, in its scope note, limited the split, leaving out thrown particles, massive fragments, splits off the equatorial plane and multi-stage splits; this paper sits in the seat of thrown particles (no earlier occupant; the massive-fragment seat was taken by Paper 995). Paper 939 showed that the formula for the gain in one split does not ask whether the falling particle can get there. Here, under the same local conditions, the energy at infinity is changed. Paper 995 showed that massive fragments lower the share extracted in one split, and that mass on the infalling fragment weighs more. Here the fragments stay photons and the splitting particle is changed. Paper 887 showed that the total energy extractable from a rotating black hole is at most 0.292893 of its mass. Here one split, not the total, is counted. What is added: finding extracted amounts and shares for four radii and six energies at infinity, confirming that the amount matches one form, confirming at every radius that harder throws extract less, giving the rate of decrease at large E_0 , and placing the separator on "energy far away, or depth of the split".

1. Notation and the Question

Split: with the gravitational constant, the speed of light and the hole's mass set to 1, on the equatorial plane of a Kerr spacetime with spin $a = 1$, a particle of mass 1 is thrown in with energy at infinity $E_0 \geq 1$ and split into two photons at its turning point at radius r . The turning condition is

$$[E(r^2 + a^2) - aL]^2 = \Delta[\mu^2 r^2 + (L - aE)^2], \quad \Delta = r^2 - 2r + a^2$$

with $\mu = 1$ for the splitting particle and $\mu = 0$ for the photons. Solutions with $E_1 + E_2 = E_0$, $L_1 + L_2 = L_0$, both photons future-directed and the infalling photon's energy E_2 negative were sought numerically, taking the largest extracted amount $E_1 - E_0$. $E_0 = 1, 1.1, 1.5, 2, 5, 10$ and $r = 1.0001, 1.2, 1.5, 1.8$.

Question: when the splitting particle is thrown in faster, how does the energy extracted in one split change?

2. Amounts and Shares Extracted

Computation 2.1: extracted amount $E_1 - E_0$ (share $(E_1 - E_0)/E_0$ in parentheses).

E_0	$r = 1.0001$	$r = 1.2$	$r = 1.5$	$r = 1.8$
1	0.207071	0.145497	0.077350	0.027046
1.1	0.193270 (0.175700)	0.134957 (0.122689)	0.071155 (0.064686)	0.024698 (0.022453)
1.5	0.151360 (0.100907)	0.103913 (0.069275)	0.053638 (0.035758)	0.018295 (0.012197)
2	0.118012 (0.059006)	0.080123 (0.040062)	0.040833 (0.020416)	0.013794 (0.006897)
5	0.049500 (0.009900)	0.033114 (0.006623)	0.016611 (0.003322)	0.005549 (0.001110)
10	0.024933 (0.002493)	0.016639 (0.001664)	0.008326 (0.000833)	0.002777 (0.000278)

The values at $E_0 = 1$ equal the photon closed form $(\sqrt{2/r} - 1)/2$, the split of Paper 939. At every radius, raising E_0 kept lowering the extracted amount.

3. The Form of the Extracted Amount

Computation 3.1: at 24 points the largest difference between the extracted amount and

$$E_1 - E_0 = \frac{1}{2} \left(\sqrt{E_0^2 - 1 + \frac{2}{r}} - E_0 \right)$$

was 9.030×10^{-8} . At $E_0 = 1$ this form returns to $(\sqrt{2/r} - 1)/2$.

Computation 3.2: putting large E_0 into this form, at $r = 1.0001$ with $E_0 = 100$ and 1000 the extracted amounts are 2.499438×10^{-3} and 2.499499×10^{-4} , and E_0 times the amount is 0.249944 and 0.249950 ; at $r = 1.5$ it is 0.083333 , approaching $(2/r - 1)/4$ (0.249950 and 0.083333).

Observation 3.3

adding energy at infinity did not increase the amount extracted. The splitting particle brings in more energy E_0 , but the escaping photon carries out only $E_1 - E_0$ more than E_0 : that difference was smaller the larger E_0 , falling in inverse proportion at large E_0 . The form was not derived, so why it falls is not stated here.

4. The Separator

Observation 4.1

the separator is whether energy is added far away or the split point is made deeper. For every E_0 , moving the split point from $r = 1.8$ to just outside the horizon increased the amount, from 0.013794 to 0.118012 at $E_0 = 2$: adding energy far away decreased it at every radius. What increased the amount extracted was the depth of the split, not the energy at infinity.

Observation 4.2

what is not said. Whether the thrown particle reaches the split point, extraction by collisions of two particles, combination with massive fragments (Paper 995), splits off the equatorial plane, multi-stage splits, derivation of the form, and real astronomical objects or jets are not treated. What is shown is how, in one split solved under local conditions, adding energy at infinity changes the amount extracted.

5. Roll Call and Closing

1. splitting a thrown particle (Section 1): energy at infinity E_0 varied.
2. amount **0.024933** at $E_0 = 10$ (Computation 2.1): down from **0.207071** at $E_0 = 1$.
3. share down to **0.002493** (Computation 2.1): falls faster still.
4. difference **9.030×10^{-8}** from the form (Computation 3.1): agreement at 24 points.
5. E_0 times the amount approaches $(2/r \text{ minus } 1)/4$ (Computation 3.2): shrinks inversely.
6. energy far away, or depth of the split (Observation 4.1): the separator; the scope note of Paper 939.

Closing: Splitting a particle thrown in faster extracts no more energy, and the harder it is thrown, the less. Raising the energy at infinity from 1 to 10 in the equatorial Kerr split of Paper 939, the amount extracted just outside the horizon fell from 0.207071 to 0.024933 and the share to 0.002493: the amount agreed at 24 points with one half of (the root of E_0 squared minus 1 plus $2/r$, minus E_0) within 9.030×10^{-8} , and at large E_0 , E_0 times the amount approached $(2/r \text{ minus } 1)/4$. Making the split deeper increased the amount for every E_0 . The separator is whether energy is added far away or the split point is made deeper. Placed among the earlier papers: to the one split Paper 939 counted with a particle at rest far away, the thrown particles Paper 939 did not treat were added, and the fall Paper 995 saw on the fragment side was seen on the side of the splitting particle. To be honest: numbers for splits

solved under local conditions were set out and checked against one form. The form was not derived. How throwing changes the amount extracted, and no further.

References

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