

The Sharpe Ratio Belongs to the Interval You Measured It On

[F032]

On the same series, measuring hourly rather than weekly moves the ratio of annualized Sharpe ratios from 0.8808 to 1.4094 — what moves is only the denominator, since the mean times bars per year stays at 0.060000 to 0.060657 while the standard deviation times $\sqrt{\text{bars}}$ per year runs from 0.084779 to 0.091200 — and the ratio matches $1 \div \sqrt{\text{variance ratio}}$ to a median of -0.000832 , so that agreement is a check, not a discovery

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Abstract

What this paper asks is whether the annualized Sharpe ratio depends on the interval it was measured on — and the answer is: it does, and the dependence can be written as an identity. No new mathematical theorem and no new law is claimed.

First, the same series gives a different annualized Sharpe at a different interval. The hourly-over-weekly ratio runs from 0.8808 to 1.4094 (median 0.9375) (Section 3).

Second, ★ what moves is only the denominator. The mean times bars per year stays at 0.060000 to 0.060657, while the standard deviation times $\sqrt{\text{bars}}$ per year runs from 0.084779 to 0.091200 (Section 5).

Third, and this is the core. ★ The gap can be written with the variance ratio. The difference between the Sharpe ratio and $1 \div \sqrt{\text{variance ratio}}$ has a median of -0.000832 . Annualizing assumes returns are independent, and the variance ratio is exactly the quantity that measures whether that assumption holds (Section 4).

Fourth, **the agreement is a check, not a discovery**. It is close to an identity, so **it is a failure to agree that should make one suspect the arithmetic** (Section 4).

Fifth, **but the ratio breaks when the denominator is near zero**. GBPUSD always-buy has annualized Sharpes of **-0.0320** and **-0.0227**, essentially zero, so its ratio of **0.7095** means nothing and the difference is **-0.271083** (Section 4).

Sixth, **★ and on a synthetic random walk the ratio still scatters from 0.8689 to 1.0422**. The measured variance ratio has a median of **0.9493**, below **1**, but **the random walk drops to 0.8001** too, so this paper does not claim mean reversion (Section 6).

Scope of this paper: No new mathematical theorem and no new law is claimed — the Sharpe ratio, the variance ratio and the $\sqrt{}$ rule for annualizing are all known. **No real course or its proponent is discussed** — no real teaching material, author, or vendor is the subject here, and none is named. **Trading is not discussed** — no costs are included, so the figures here are not trading performance (Section 7). **The Sharpe ratio = $1 \div \sqrt{\text{variance ratio}}$ is close to an identity** — the agreement is a check, not a discovery (Section 4). **Mean reversion is not claimed** — the measured variance ratio falls below 1, but the synthetic random walk falls just as far (Section 6). **No risk-free rate is subtracted** — the Sharpe ratio here is mean $\div$ standard deviation (Section 7). **Four intervals are fixed** — one hour, four hours, one day, one week (Section 2). **The weekly sample is small** — 290 to 304 observations per instrument, so the variance ratio is unstable there (Section 7). **The scope is intraday and few instruments** — four instruments, six years, one-hour bars. Monthly time-series momentum (dozens of markets, one to twelve month holds) is documented in the academic literature and is not contradicted here. **No forecast is claimed**.

1. Notation and the question

A single annualized Sharpe ratio is often reported and left at that.

What gets reported	What does not
The annualized Sharpe ratio	Which interval's returns were stretched by the $\sqrt{\cdot}$
The period and the instrument	Whether the independence assumption was checked
The annualized figure	What another interval would give

Observation 1.1: ★ Annualizing assumes returns are independent. The variance of k periods is k times the variance of one only under independence, **and only then can it be divided back by $\sqrt{k}$.**

Observation 1.2: ★ Without independence, each interval gives a different annualized Sharpe. All from the same data, and all different.

Observation 1.3: The separator is the interval the returns are measured on. The data, the period, the instrument and the series are all held fixed, and **only how many bars are summed before measuring is varied.**

Observation 1.4: ★ And the size of the gap can be written with the variance ratio: the variance of k periods $\div$ ($k \times$ the variance of one). **Under independence it is 1.**

Question: does the annualized Sharpe ratio depend on the interval it was measured on?

2. State the quantity and the design first

Predictions (written before running)

- ① If returns are independent, **every interval gives the same annualized Sharpe ratio.**
- ② If they are not, **it shifts**, and the direction is set by the variance ratio. Above 1 (positive autocorrelation) means the longer interval gives a **smaller** Sharpe.
- ③ ★ The ratio of annualized Sharpe ratios should match $1 \div \sqrt{\text{variance ratio}}$. **That is close to an identity, so agreement is a check, not a discovery.**
- ④ ★ On a synthetic random walk with zero directional information, **the variance ratio should be 1 and the ratio 1.**

Design (declared before running and not changed during it)

Item	Decision
Grid	One-hour bars, mid prices, completed bars only
Intervals	1 bar (one hour) / 4 (four hours) / 24 (one day) / 120 (one week)
Series	Always buy (log returns) / multiplied by the sign of the SMA(20) slope
Annualizing	$\sqrt{\text{bars per year}}$. Bars per year comes from the actual bar count
Measured	Annualized Sharpe ratio / variance ratio / $1 \div \sqrt{\text{variance ratio}}$
Costs	Not included. No risk-free rate subtracted
Baseline	Synthetic random walk down to within-bar movement (3 series)
Instruments	USDJPY, EURUSD, GBPUSD, XAUUSD
Period	2019–2024

Observation 2.1: ★ The sign comes from a quantity known as of the previous bar. The slope of the moving average as of bar i is used for the return from i to $i+1$. **There is no look-ahead.**

Observation 2.2: Bars per year comes from the actual bar count. FX stops at the weekend, so **rather than dividing by calendar time, the actual number of bars is counted: 5,852 to 6,094** one-hour bars per year.

Observation 2.3: When summing k bars, the remainder is dropped. At one week (120 bars) up to 119 bars are lost, **which out of 34,891 to 36,553 bars barely touches the mean.**

3. Measuring the same series on four intervals

Calculation 3.1 Annualized Sharpe by interval (always buy)

Instrument	Interval	Count	Per year	Annualized Sharpe
USDJPY	one hour	36,533	6,090	0.6672
USDJPY	one day	1,522	254	0.6600
USDJPY	one week	304	51	0.7155
EURUSD	one hour	36,553	6,094	− 0.2261
EURUSD	one week	304	51	− 0.2237
GBPUSD	one hour	36,534	6,091	− 0.0320
GBPUSD	one week	304	51	− 0.0227
XAUUSD	one hour	34,891	5,852	0.7973
XAUUSD	one day	1,453	244	0.8103
XAUUSD	one week	290	49	0.8076

Calculation 3.2 Annualized Sharpe by interval (medians across four instruments)

Series	One hour	Four hours	One day	One week	Hour ÷ week
Always buy	0.3176	0.3159	0.3143	0.3464	0.9170
Sign of the slope	0.2739	0.2827	0.2636	0.2920	0.9382

Observation 3.3: The same series gives a different annualized Sharpe at a different interval. The hourly-over-weekly ratio runs from **0.8808** to **1.4094** (median **0.9375**).

Observation 3.4: ★ XAUUSD on the sign of the slope gives **0.9053** hourly and **1.0278** weekly. The same six years and the same trading, and whether the reported figure is below or above 1 changes.

Observation 3.5: Across all 16 combinations the annualized Sharpe runs from **−0.4307** to **1.0278**. This paper does not claim those levels, since neither costs nor a risk-free rate are included.

Observation 3.6: The count collapses with the interval, from **36,533** hourly observations to **304** weekly. ★ The spread of the estimate grows by the same amount.

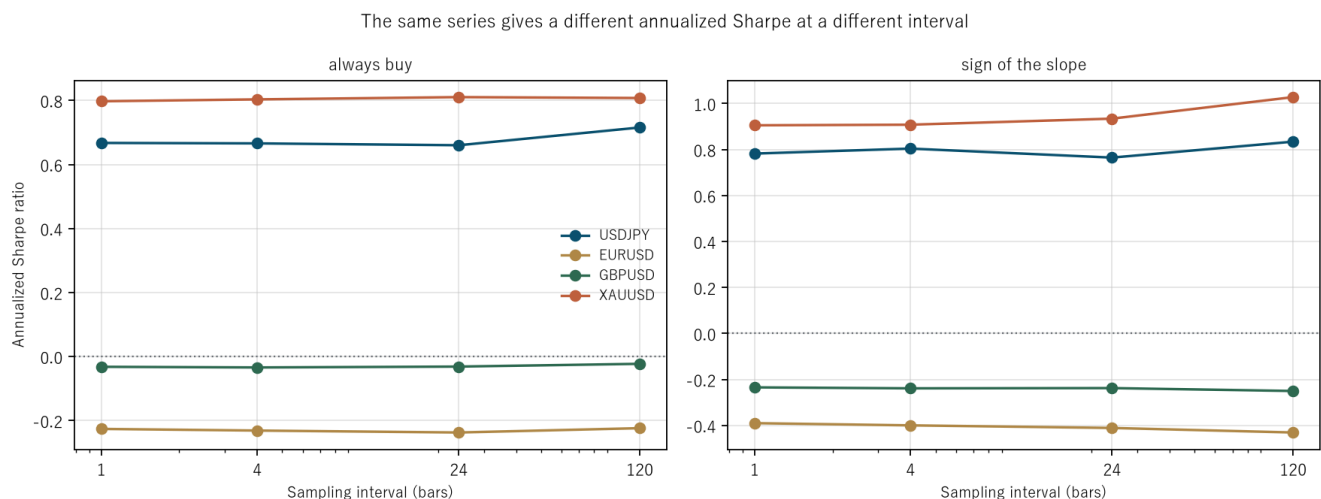


Figure 1: horizontal is the sampling interval (log scale), vertical is the annualized Sharpe ratio. **Left is always buy, right is the sign of the slope.**

4. The direction of the gap is set by the variance ratio

Calculation 4.1 Variance ratio against the Sharpe ratio

Series	Instrument	Interval	Variance ratio	$1 \div \sqrt{\text{VR}}$	Sharpe ratio	Difference
Always buy	USDJPY	four hours	1.0096	0.9952	0.9979	0.0027
Always buy	USDJPY	one day	1.0286	0.9860	0.9892	0.0032
Always buy	USDJPY	one week	0.8889	1.0607	1.0723	0.0116
Always buy	XAUUSD	one day	0.9512	1.0253	1.0163	− 0.0090
Always buy	GBPUSD	one week	1.0400	0.9806	0.7095	− 0.2711
Sign of the slope	USDJPY	one week	0.8778	1.0673	1.0662	− 0.0011
Sign of the slope	EURUSD	one week	0.7520	1.1532	1.1041	− 0.0491
Sign of the slope	GBPUSD	one day	0.9906	1.0047	1.0134	0.0087
Sign of the slope	XAUUSD	four hours	0.9948	1.0026	1.0029	0.0003
Sign of the slope	XAUUSD	one week	0.7634	1.1445	1.1354	− 0.0092

Observation 4.2: ★ This is the core. The difference between the Sharpe ratio and $1 \div \sqrt{\text{variance ratio}}$ has a median of -0.000832 . **The gap can be written with the variance ratio.**

Observation 4.3: The agreement is a check, not a discovery. Annualizing is the act of dividing by $\sqrt{k}$, and the variance ratio measures the departure from that k-fold growth, so **the two write the same thing from opposite sides.** ★ **It is a failure to agree that should make one suspect the arithmetic.**

Observation 4.4: And one case did fail to agree. GBPUSD always-buy at one week differs by -0.2711 . ★ **Its annualized Sharpes are -0.0320 and -0.0227 , essentially zero, so the ratio of 0.7095 means nothing. When the denominator is near zero, the ratio breaks.**

Observation 4.5: The variance ratio runs from 0.7520 to 1.0521 (median 0.9493), above 1 in 4 of 24 cases. Most fall below 1 — variance growing less than k-fold — but whether that is mean reversion is settled in Section 6.

Observation 4.6: ★ Annualizing makes an assumption: that returns are independent, so the variance of k periods is k times the variance of one. **The variance ratio is exactly the quantity that measures whether that assumption holds.**

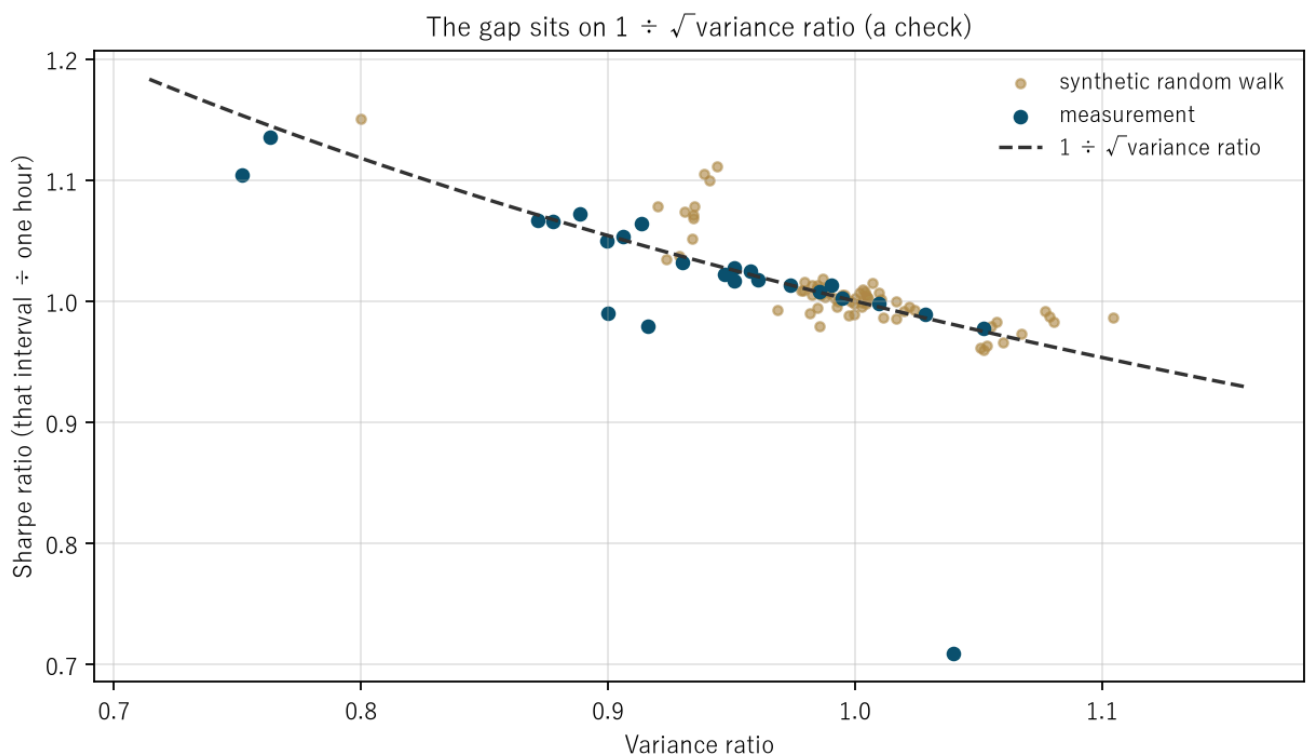


Figure 2: horizontal is the variance ratio, vertical is the Sharpe ratio (that interval ÷ one hour). **The dashed line is $1 \div \sqrt{\text{variance ratio}}$; points on it mean the identity holds.**

5. Is it the numerator that moves, or the denominator?

Calculation 5.1 Mean and standard deviation, printed as they are (USDJPY)

Series	Interval	Mean	Standard deviation	Mean × per year	SD × $\sqrt{\text{per year}}$
Always buy	one hour	0.00000985	0.00115227	0.060000	0.089924
Always buy	four hours	0.00003951	0.00231560	0.060163	0.090355
Always buy	one day	0.00023720	0.00572508	0.060194	0.091200
Always buy	one week	0.00119514	0.01190036	0.060657	0.084779
Sign of the slope	one hour	0.00001155	0.00115195	0.070327	0.089899
Sign of the slope	four hours	0.00004630	0.00224689	0.070489	0.087675
Sign of the slope	one day	0.00027790	0.00578841	0.070521	0.092209
Sign of the slope	one week	0.00138425	0.01182304	0.070254	0.084229

Observation 5.2: ★ The mean times bars per year is essentially the same at every interval: 0.060000 to 0.060657 for always buy, 0.070254 to 0.070521 for the sign of the slope.

Observation 5.3: ★ That is as it must be. The sum of log returns does not depend on how many are grouped together, so if this moved, the arithmetic would be broken.

Observation 5.4: ★ What moves is the standard deviation times $\sqrt{bars}$ per year: 0.084779 to 0.091200 for always buy, 0.084229 to 0.092209 for the sign of the slope.

Observation 5.5: ★ So the annualized Sharpe moves because of the denominator. The same profit is divided by a different amount of "risk".

Observation 5.6: And which interval is right is not settled here. ★ What settles it is the time scale on which the account is watched, not the data. Someone who looks at the account daily and someone who looks weekly face a different size of risk from the same trading.

6. Baseline — do the same thing on a series with zero directional information

Calculation 6.1 Measurement against synthetic random walk (medians across four instruments)

Series	Interval		Variance ratio	Annualized Sharpe
Always buy	one hour	measured	1.0000	0.3176
Always buy	one hour	random	1.0000	0.4494
Always buy	one day	measured	0.9337	0.3143
Always buy	one day	random	1.0008	0.4532
Always buy	one week	measured	0.9369	0.3464
Always buy	one week	random	1.0514	0.4409
Sign of the slope	one day	measured	0.9604	0.2636
Sign of the slope	one day	random	0.9875	0.4896
Sign of the slope	one week	measured	0.8174	0.2920
Sign of the slope	one week	random	0.9401	0.4786

Observation 6.2: Prediction ④ held at the median. The random walk's variance ratio has a median of **0.9958** (the theoretical value is 1) and its hourly-over-weekly ratio a median of **0.9834**.

Observation 6.3: But the random walk scatters widely too. Its variance ratio runs from **0.8001** to **1.1044** and its ratio from **0.8689** to **1.0422**. ★ That a series built independent scatters this much is because the weekly sample is only 290 to 304 observations.

Observation 6.4: ★ So the measured variance ratio falling below 1 (median 0.9493) cannot be taken as evidence of mean reversion. The random walk falls to 0.8001 as well. That this baseline has no magnitude structure was measured in [F033].

Observation 6.5: The measured ratio's median of 0.9375 is lower than the random walk's 0.9834.

★ That may be a sign of mean reversion, but the ranges overlap, so this paper does not claim it as an effect.

Observation 6.6: ★ The identity side, by contrast, holds equally on both. What this paper claims is that side: the annualized Sharpe depends on the interval, and the dependence can be written with the variance ratio.

7. Limitations

1. ⚠ **The Sharpe ratio = $1 \div \sqrt{\text{variance ratio}}$ is close to an identity.** The agreement is a check, not a discovery. **This paper says so.**
 2. **The ratio breaks when the denominator is near zero.** GBPUSD always-buy has a Sharpe of essentially zero, so its ratio of **0.7095** means nothing.
 3. **Mean reversion is not claimed.** The measured variance ratio falls below 1, but the synthetic random walk falls just as far.
 4. **The weekly sample is small: 290 to 304** observations per instrument, so the variance ratio is unstable there.
 5. **No risk-free rate is subtracted.** The Sharpe ratio here is mean $\div$ standard deviation.
 6. **No costs are included.** The figures here are therefore not trading performance.
 7. **Four intervals are fixed,** and the remainder is dropped when summing.
 8. **Four instruments over six years.** Nothing is said about other markets or other periods.
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Conclusion

The same series was measured on four intervals: one hour, four hours, one day and one week. The ratio of annualized Sharpe ratios (hour over week) runs from **0.8808** to **1.4094**, with a median of **0.9375**. ★

XAUUSD on the sign of the slope gives 0.9053 hourly and 1.0278 weekly: the same six years and the same trading, and whether the reported figure is below or above 1 changes.

★ **What moves is only the denominator.** The mean times bars per year stays at **0.060000** to **0.060657**, because the sum of log returns does not depend on how many are grouped together (if it moved, the arithmetic would be broken). **What moves is the standard deviation times $\sqrt{\text{bars}}$ per year (0.084779 to 0.091200).**

The same profit is divided by a different amount of "risk".

★ **And the gap can be written with the variance ratio.** The difference between the Sharpe ratio and $1 \div \sqrt{\text{variance ratio}}$ has a median of **-0.000832**. **The agreement is a check, not a discovery.** Annualizing assumes returns are independent so that the variance of k periods is k times the variance of one, and **the variance ratio is exactly the quantity that measures whether that holds.**

One case did fail to agree. GBPUSD always-buy at one week differs by **-0.2711**, because its annualized Sharpes of **-0.0320** and **-0.0227** are essentially zero and the ratio of **0.7095** means nothing. When the denominator is near zero, the ratio breaks.

The measured variance ratio has a median of 0.9493, below 1, but this paper does not claim mean reversion, because the synthetic random walk falls to 0.8001 as well. With only 290 to 304 weekly observations, even an independently built series scatters this much.

★ **So a report of an annualized Sharpe ratio needs the interval it was measured on. And which interval is right is not settled by the data. What settles it is the time scale on which the account is watched. Someone who looks daily and someone who looks weekly face a different size of risk from the same trading.**

Data and reproduction

Data	FX is Dukascopy-derived tick data (from 2019 onward, confirmed identical down to the millisecond, bid and ask). Gold is Dukascopy
Period	2019–2024 (six years)
Instruments	USDJPY, EURUSD, GBPUSD, XAUUSD
Bars	One-hour bars, mid prices, closes of completed bars only. No bar is made where there are no ticks
Intervals	1, 4, 24 and 120 bars (one hour, four hours, one day, one week). The remainder is dropped
Series	Always buy (log returns) / multiplied by the sign of the SMA(20) slope
Annualizing	$\sqrt{\text{bars per year}}$, with bars per year from the actual bar count
Costs	Not included. No risk-free rate subtracted
Verification	<code>検算/F032.py</code> (<code>--zu</code> also writes the figures). Every figure in the text comes from <code>検算/F032_output.txt</code>
Figures	<code>図/F032_zu1</code> , <code>図/F032_zu2</code> (png, svg, English versions)
Random seeds	Fixed. Re-running gives the same values
Foundation check	<code>_ツール/auditBars.py</code> (bar look-ahead, daylight saving, day boundaries). The record is <code>_土台の検査.txt</code>

References

- Yamagishi, Y. (2026). A Synthetic Random Walk Has No Volatility Clustering [F033]. This series, unpublished