

The Effective Count of a Diversified Book Is What Is Left After the Pair Correlations Cancel [F048]

The six pair correlations among four instruments run from -0.4252 to $+0.6577$, yet averaging them gives $+0.0455$ and an effective count of 3.5194 , which looks almost independent — and split two at a time, the same "two" opens from 1.2065 to 3.4797

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Abstract

What this paper asks is what actually sets the reduction in risk behind "split across several currency pairs and the risk goes down". No new mathematical theorem and no new law is claimed.

First, ★ the pair correlations are all over the place. EURUSD with GBPUSD is $+0.6577$ and USDJPY with EURUSD is -0.4252 ; the six pairs land on both sides of zero (Section 3).

Second, and this is the core of the paper: ★★ averaging them gives $+0.0455$, which looks almost independent. The effective count is 3.5194 , or 0.8799 of the 4 streams. "An effective count near 4" does not mean "the four were independent" (Section 4).

Third, ★★ split two at a time, the same "split in two" opens from 1.2065 to 3.4797 . EURUSD with GBPUSD gives 1.2065 — barely one stream's worth (Section 5).

Fourth, ★ the correlations move year by year, and one pair keeps climbing. EURUSD with GBPUSD goes from $+0.5152$ in 2020 to $+0.8009$ in 2023, while the effective count falls from 4.2227 to 3.3299 (Section 6).

Fifth, ★ prediction ⑤ failed. The maximum drawdown shrank by 2.2229 , better than volatility's 1.8760 (Section 7).

Sixth, ★ **splitting by hour of the day is closer to independent than splitting by instrument.** Effective count $\div$ streams is **0.8799** for instruments against **1.0008** for hours. ⚠ **Those are different hours of the same day, however, not independent trades** (Section 7).

Scope of this paper: no new mathematical theorem and no new law is claimed — the variance of an equally weighted book is an elementary result. **No real course, author, or vendor is the subject here, and none is named. No trading is discussed** — neither performance nor profit and loss is measured, and no advice on how to split is given.

Predictions ②③⑤ failed — ② put the effective count far too low (Sections 4, 6 and 7). **The hour-of-day streams are not independent trades** — they are different hours of the same day (Section 7). **Four instruments are fixed** (Section 7). **The scope is intraday and few instruments** — four instruments over six years. Results documented at longer holding periods, such as monthly time-series momentum, are not contradicted. **No prediction is claimed.**

1. Notation and the question

"Split across several currency pairs and the risk goes down," people say — usually meaning that four streams give a quarter of the risk.

Split equally n ways, the book's variance is $(1/n) \cdot \sigma^2 \cdot (1 + (n-1) \cdot \rho)$. So:

effective count = $n \div (1 + (n-1) \cdot \rho)$

What gets said	What does not get said
Four streams, so a quarter	How correlated those four were
I am diversified	The count is yours to choose; the correlation is not
The correlation is low	Low on average, or low in every pair?

Observation 1.1: ★ At $\rho = 0$ it is n streams' worth, at $\rho = 1$ one stream's worth. What does the work is the correlation, not the count.

Observation 1.2: ★ And the correlation is not yours to choose. You can set the number of streams; you cannot set how much the market moves together.

Observation 1.3: The separator here is the correlation between the pair. The instruments, the period, the grid and the way returns are taken are all held fixed, and what is looked at is not how many streams there were but how correlated they were.

Question: in actual foreign exchange, what is that correlation?

2. State the quantities and the design first

Declared before the run and unchanged during it.

Grid	1-hour bars (mid price, completed bars only)
Returns	Every 24 bars, non-overlapping , in pips
Alignment	★ The bar timestamps are aligned first, then cut into 24s
Correlation	Pearson
Effective count	$n \div (1 + (n - 1) \cdot \rho^-)$ (formula) and $\sigma^2(\text{one stream}) \div \sigma^2(\text{book})$ (measured)
Book	Equally weighted. Instruments are scaled to equal volatility
Windows	Calendar years 2019–2024 / hours UTC 0-7, 8-15, 16-23
Baseline	Synthetic random walk down to within the bar (3 draws, an independent seed per instrument)
Separator	The correlation between the pair (not how many streams)
Instruments and period	USDJPY, EURUSD, GBPUSD, XAUUSD / 2019–2024

Observation 2.1: ★ The order of the alignment is the point. The instruments do not share the same set of bars, so cutting into 24s first leaves the boundaries out of step. Cutting first made every correlation NaN.

Observation 2.2: Returns are taken every 24 bars, leaving 1,395 aligned points. The volatilities are **78.3978** for USDJPY, **48.5303** for EURUSD, **72.6165** for GBPUSD and **178.4380** pips for XAUUSD. **★ The hourly series hold 36,534, 36,554, 36,535 and 34,892 bars, so the aligned points are fewer than any single instrument's.**

Observation 2.3: ★ That the interval at which returns are measured moves the Sharpe ratio was measured in [F032]. This paper fixes that interval at 24 bars and does not move it.

Predictions (written before measuring)

- ① Pairs sharing a currency are highly correlated, so the effective count does not grow.
- ② ★ Split equally across four instruments, the effective count is not 4 but around 2.
- ③ ★★ The correlation moves over time; measured window by window, the effective count changes a great deal.
- ④ A synthetic random walk (built independently) gives an effective count equal to the number of streams.
- ⑤ ★ The maximum drawdown shrinks **worse** than the variance does.
- ⑥ ★ Splitting by hour of the day is less correlated than splitting by instrument.

3. The pair correlations run from -0.4252 to $+0.6577$

Calculation 3.1 The six pairs

Pair	Correlation
EURUSD with GBPUSD	+0.6577
USDJPY with EURUSD	− 0.4252
EURUSD with XAUUSD	+0.3797
USDJPY with XAUUSD	− 0.3325
USDJPY with GBPUSD	− 0.3284
GBPUSD with XAUUSD	+0.3218

Observation 3.2: Prediction ① held. ★ The highest is EURUSD with GBPUSD at +0.6577, a pair with the dollar on the same side in both.

Observation 3.3: ★ All three pairs containing USDJPY are negative (−0.4252, −0.3325, −0.3284), because the dollar sits on the other side.

Observation 3.4: ★★ Three positive, three negative — and of similar size. That is what matters in the next section.

4. Averaged, they cancel, and the effective count comes out at 3.5194

Calculation 4.1 Split equally four ways

Quantity	Value
Streams n	4
Mean pair correlation ρ^-	+0.0455
Effective count (formula $n \div (1 + (n - 1)\rho^-)$)	3.5194
Effective count (measured $\sigma^2(\text{one}) \div \sigma^2(\text{book})$)	3.5194
Difference between formula and measurement	0.000000

Observation 4.2: ★★ Prediction ② failed. The effective count is not around 2 but 3.5194. That is 0.8799 of the 4 streams — far closer to 4 than I expected.

Observation 4.3: ★★ The reason is cancellation. +0.6577 and −0.4252 mix, and the average comes out at +0.0455.

Observation 4.4: ★★ So "an effective count near 4" does not mean "the four were independent". There is a +0.6577 pair inside. A low average and a low correlation in every pair are different

things.

Observation 4.5: The formula and the measurement agree exactly (difference 0.000000). ★ That is a check, not a finding. It merely confirms that an equally weighted book's variance is $(1/n) \cdot \sigma^2 \cdot (1 + (n-1)\rho)$.

Observation 4.6: Risk (volatility) shrinks by $\sqrt{3.5194} = 1.8760$. ★ Split four ways, it does not halve.

5. Split two at a time, it opens from 1.2065 to 3.4797

Calculation 5.1 The effective count of each pair

Pair	Correlation	Effective count
USDJPY with EURUSD	− 0.4252	3.4797
USDJPY with XAUUSD	− 0.3325	2.9961
USDJPY with GBPUSD	− 0.3284	2.9780
GBPUSD with XAUUSD	+0.3218	1.5130
EURUSD with XAUUSD	+0.3797	1.4496
EURUSD with GBPUSD	+0.6577	1.2065

Observation 5.2: ★★ The same "split in two" opens from 1.2065 to 3.4797. The median is 2.2455.

Observation 5.3: ★★ EURUSD with GBPUSD gives 1.2065 — barely one stream's worth. Two positions are on, and the risk falls by only $\sqrt{1.2065}$.

Observation 5.4: ★★ Negatively correlated pairs, on the other hand, exceed 2. USDJPY with EURUSD gives 3.4797. Split in two, it works like three and a half.

Observation 5.5: ★ So "how many I split it into" says nothing about the effective count. The same 2 reads as 1.2065 or as 3.4797.

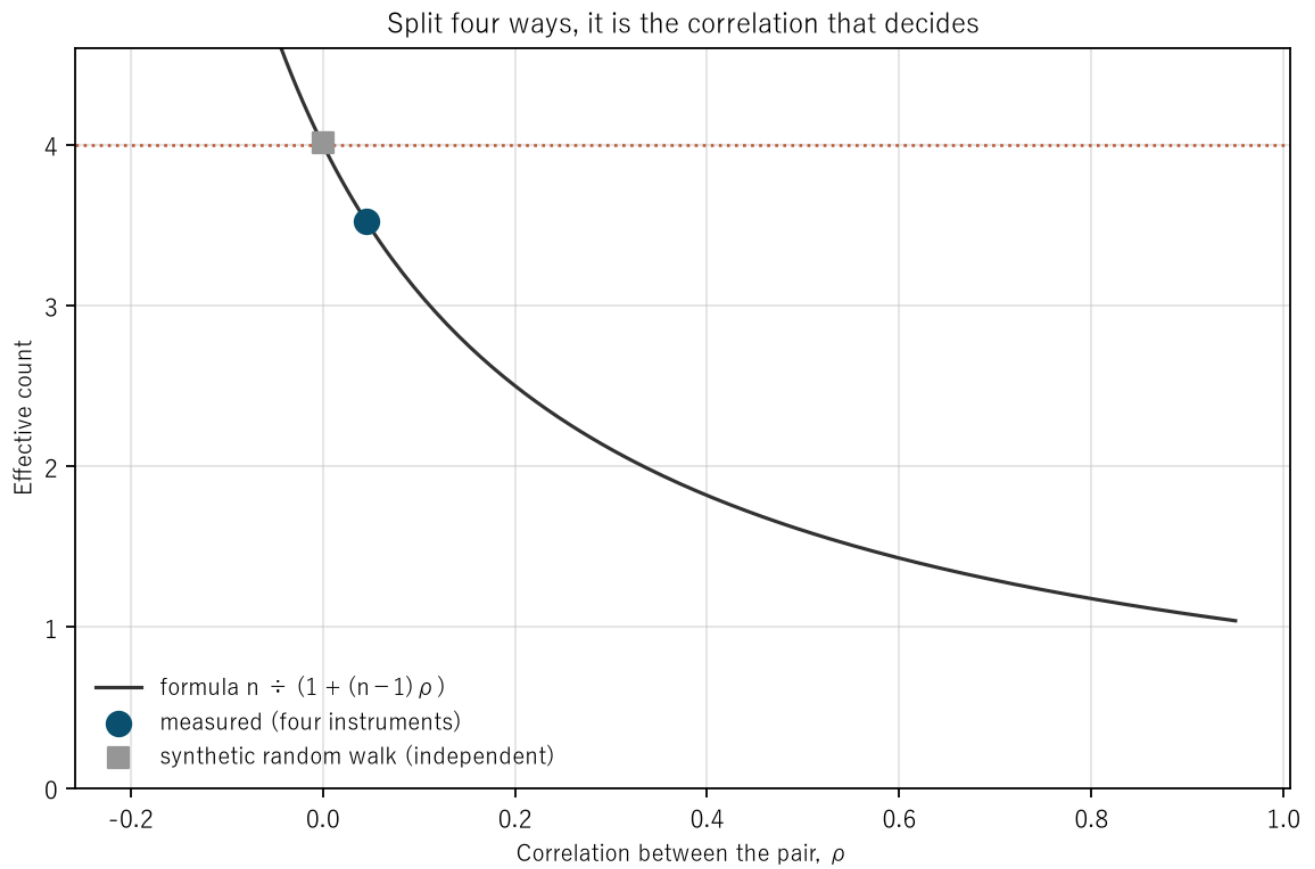


Figure 1: the pair correlation ρ on the horizontal axis and the effective count on the vertical. **The curve is the formula $n \div (1 + (n-1)\rho)$, and the points are the measurement and the random walk.**

6. The correlations move year by year, and one pair keeps climbing

Calculation 6.1 By calendar year

Year	Mean pair correlation	Effective count	Highest pair
2019	−0.0176	4.2227	USDJPY/XAUUSD −0.5660
2020	+0.0159	3.8181	EURUSD/GBPUSD +0.5152
2021	+0.0603	3.3871	EURUSD/GBPUSD +0.6840
2022	+0.0530	3.4514	EURUSD/GBPUSD +0.7524
2023	+0.0437	3.5367	EURUSD/GBPUSD +0.8009
2024	+0.0671	3.3299	EURUSD/GBPUSD +0.7663

Observation 6.2: ★ Prediction ③ failed. The effective count runs from 4.2227 to 3.3299, a maximum ÷ minimum of only 1.2681. I wrote "changes a great deal", and it does not.

Observation 6.3: ★★ The contents move, though. EURUSD with GBPUSD climbs from +0.5152 in 2020 to +0.8009 in 2023. The average stays put because the other pairs move the other way and cancel it.

Observation 6.4: ★ 2019 is the one year whose highest pair is USDJPY with XAUUSD at −0.5660. That year the average is −0.0176 and the effective count reaches 4.2227, above 4.

Observation 6.5: ★ Watching only the average, you would not notice a +0.8009 pair growing. That is the same shape as Section 4.

7. The tail, the hours, the baseline and limitations

Calculation 7.1 How the maximum drawdown shrinks

Quantity	Value
Maximum drawdown of one stream (median)	38.8708
Maximum drawdown of the four combined	17.4867
Shrink factor	2.2229
Volatility's shrink, $\sqrt{\text{effective count}}$	1.8760
Tail $\div$ volatility	1.1849

Observation 7.2: ★ Prediction ⑤ failed. The tail shrank better (2.2229 against 1.8760, a ratio of 1.1849). ★ That the maximum drawdown belongs to the order of the trades was measured in [F031], however, so this single number should not be read hard.

Calculation 7.3 Splitting by instrument or by hour

Split by	Streams	Mean pair correlation	Effective count	Effective $\div$ streams
Instrument (four)	4	+0.0455	3.5194	0.8799
Hour of the day (three)	3	− 0.0004	3.0023	1.0008

Observation 7.4: Prediction ⑥ held. The three hour-block correlations are −0.0002, −0.0008 and −0.0002 — essentially zero.

Observation 7.5: ⚠ Those are different hours of the same day, however. They are not independent trades.

Calculation 7.6 The baseline (synthetic random walk, independent per instrument)

Seed	Mean pair correlation	Effective count (formula)	Effective count (measured)
0	− 0.0018	4.0213	4.0213
1	− 0.0011	4.0129	4.0129
2	+0.0005	3.9941	3.9941

Observation 7.7: Prediction ④ held. The median is 4.0129, matching the 4 streams. ★ That too is a check, not a finding.

Observation 7.8: ★ The measured 3.5194 is 0.8770 of it. So the four instruments do the work of 0.8770 of four independent streams.

Observation 7.9: ★ What this baseline is zero about was measured in [F033].

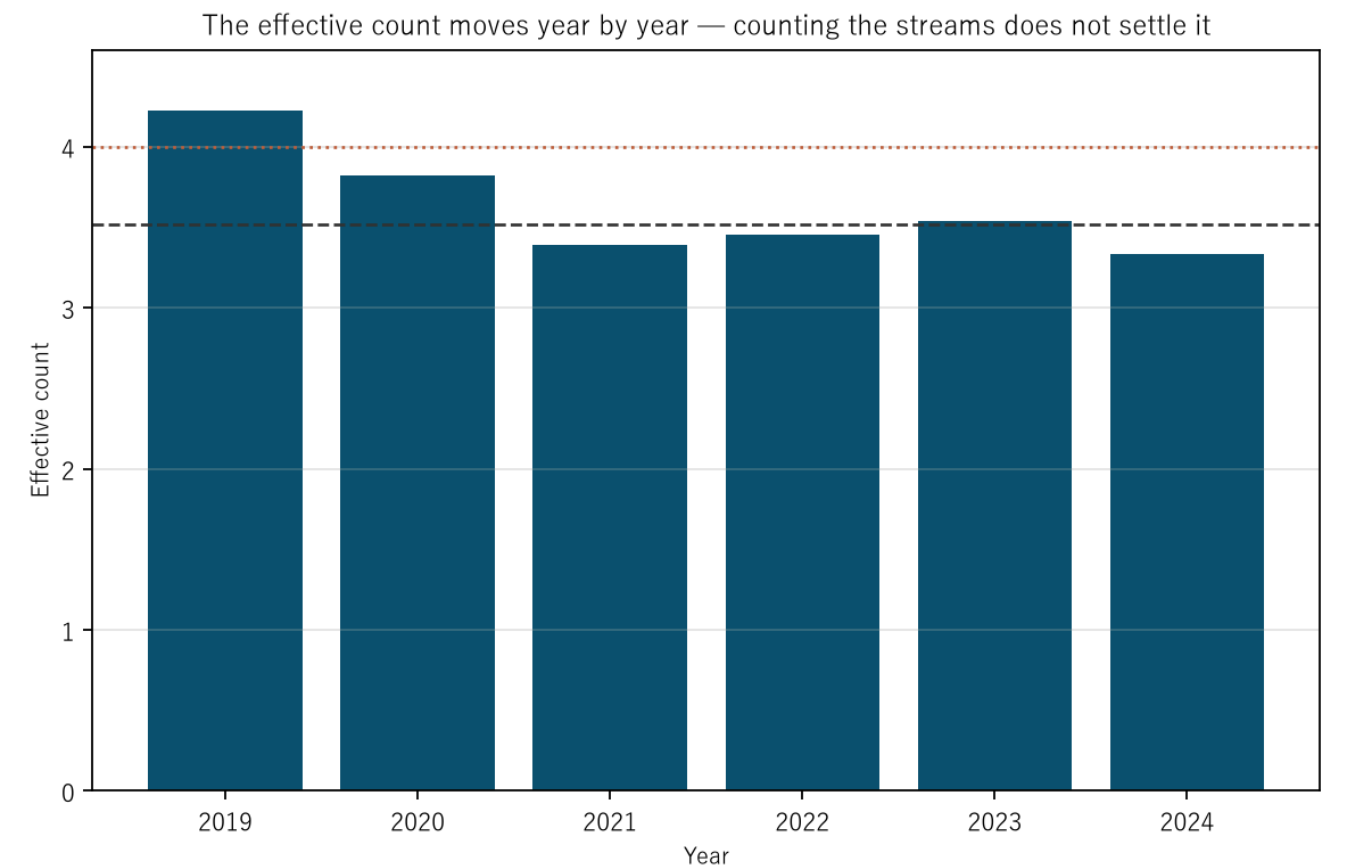


Figure 2: year on the horizontal axis and the effective count on the vertical. **The dotted line is the 4 streams and the dashed line the 3.5194 of the whole period.**

Limitations

1. **⚠ Prediction ② was badly wrong.** I put the effective count at around 2; it is 3.5194. **I had not thought about the cancellation.**
2. **Predictions ③ and ⑤ failed too.** ③ overstated the year-to-year movement and ⑤ got the tail's shrinking backwards.
3. **The hour-of-day streams are not independent trades.** They are different hours of the same day.
4. **Four instruments are fixed.** A different set gives a different average.
5. **The return interval is fixed at 24 bars.**
6. **No trading is discussed.** Neither performance nor profit and loss is measured, and no advice on how to split is given.
7. **Four instruments, six years.** Nothing is claimed about other markets or other periods.

Conclusion

When someone says "four ways, so a quarter", what is being counted is the number of streams. **But what the formula contains is the correlation.**

★ The six pair correlations ran from +0.6577 to −0.4252. Three positive, three negative, and of similar size.

★★ So averaged they give +0.0455 and look almost independent. The effective count is 3.5194, 0.8799 of the 4 streams. I predicted around 2 and was wrong.

★★ But "an effective count near 4" does not mean "the four were independent". There is a +0.6577 pair inside. A low average and a low correlation in every pair are different things.

★★ Split two at a time, that shows plainly. The same "split in two" opens from 1.2065 to 3.4797. EURUSD with GBPUSD gives 1.2065, barely one stream's worth; USDJPY with EURUSD gives

3.4797, three and a half.

★ Year by year the average barely moves (4.2227 to 3.3299, a factor of 1.2681), while EURUSD with GBPUSD climbs from +0.5152 to +0.8009. Watching only the average, you would not notice the pair that is growing.

★ Three of six predictions failed. ② put the effective count too low, ③ overstated the year-to-year movement and ⑤ got the tail backwards — **it shrank better (2.2229 against 1.8760).**

So what should be written down when claiming diversification is not the number of streams. ★ It is the list of pair correlations, their average, and the highest one. The average is what cancellation leaves behind, and by itself it does not show what is inside.

Data and reproduction

Data	Foreign exchange from Dukascopy-derived ticks (from 2019 onward, matching to the millisecond, bid and ask, has been verified). Gold from Dukascopy
Period	2019–2024 (six years)
Instruments	USDJPY, EURUSD, GBPUSD, XAUUSD
Bars	1-hour, mid price, completed bars only. Bars with no ticks are not created
Returns	Every 24 bars, non-overlapping, in pips
Alignment	The bar timestamps are aligned across the four instruments first, then cut into 24s
Effective count	$n \div (1 + (n - 1) \cdot \rho^-)$ (formula) and $\sigma^2(\text{one stream}) \div \sigma^2(\text{book})$ (measured); ρ^- is the mean pair correlation
Book	Equally weighted. Instruments are scaled to equal volatility
Windows	Calendar years 2019–2024 / hours UTC 0-7, 8-15, 16-23
Baseline	Synthetic random walk down to within the bar (3 draws), with an independent seed per instrument
Verification	<code>検算/F048.py</code> (<code>--zu</code> also writes the figures). Every number in the text comes from <code>検算/F048_output.txt</code>
Figures	<code>図/F048_zu1</code> , <code>図/F048_zu2</code> (png, svg, English versions)
Random seeds	Fixed. Re-running gives the same values
Substrate check	<code>_ツール/audit_bars.py</code> (bar look-ahead, daylight saving, day boundary). The record is <code>_土台の検査.txt</code>

References

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