The Fat Tails and Volatility Clustering of Minute Bars Are Half an Artefact of the Clock [F050]

A synthetic random walk whose true values are normal and uncorrelated shows an excess kurtosis of 2.2459 and an autocorrelation of 0.1812 when cut into minute bars, and returns to −0.0007 and −0.0001 when cut by tick count — on the real data the kurtosis falls from 120.2788 to 61.3744 and the autocorrelation from 0.3077 to 0.1767

山岸 勇輝（Yuuki YAMAGISHI）／Independent Researcher
ORCID 0009-0006-8587-3450
September 2026／CC BY 4.0

Abstract

What this paper asks is how much the shape of returns is made by what a bar was cut on. No new mathematical theorem and no new law is claimed.

First, ★ **what was held constant and what was left to chance swap over cleanly.** Minute bars vary by only **1.31** in duration but by **10.38** in tick count; tick bars vary by **1.00** in tick count (by construction) and by **7.04** in duration (Section 3).

Second, ★★ **the fat tail halves with the cut.** Excess kurtosis is **120.2788** for minute bars and **61.3744** for tick bars, a factor of **1.9598** (Section 4).

Third, ★★ **volatility clustering halves too.** The lag-1 autocorrelation of \|return\| goes from **0.3077** to **0.1767**, a factor of **0.5743** (Section 5).

Fourth, and this is the core of the paper: ★★ **the baseline settles it. A synthetic random walk whose true values are normal and uncorrelated shows an excess kurtosis of 2.2459 and an autocorrelation of 0.1812 when cut into minute bars. Cut by tick count it returns to −0.0007 and −0.0001** (Section 7).

Fifth, ★ **the hour-of-day bias is not absorbed; it is brought out as a count.** Minute bars vary by only **1.0647** across hours, while tick bars vary by **6.7105** (Section 6).

Sixth, ★ **the spread of the moves was already measured in [F018]. What this paper adds is the kurtosis, the autocorrelation and the baseline** (Sections 4 and 7).

Scope of this paper: **no new mathematical theorem and no new law is claimed** — time change and subordinated processes are classical subjects. **No real course, author, or vendor is the subject here, and none is named. No trading is discussed** — neither performance nor profit and loss is measured. **The spread of the moves overlaps [F018]** — what this paper adds is the kurtosis, the autocorrelation and the baseline (Section 4). **Two ways of cutting are fixed** — the clock and the tick count (Section 7). **No bar is created for a slot with no ticks** — that this convention moves the answer was measured in [F049] (Section 7). **The scope is intraday and few instruments** — four instruments over six years. Results documented at longer holding periods, such as monthly time-series momentum, are not contradicted. **No prediction is claimed.**

---

1. Notation and the question

To make a bar, something has to be held constant.

Held constant	Left to chance	Called
Time	The number of ticks inside a bar	A minute bar
The number of ticks	The time a bar takes	A tick bar

Observation 1.1: ★ **Both are called "a bar", and what was held constant differs.**

Observation 1.2: ★ **That bars cut by tick count are a different clock from bars cut by the clock was measured in [F018]. What was measured there was the spread of the moves. What this paper measures is the shape of the returns.**

Observation 1.3: **The separator here is what the bar was cut on.** The instruments, the period and the sequence of ticks are all held fixed, and **only the way of cutting is changed.**

Question: how much of the fat tail and the volatility clustering does the way of cutting make?

---

2. State the quantities and the design first

Declared before the run and unchanged during it.

A, minute bars	Every 60,000 milliseconds. **No bar is created for a slot with no ticks**
B, tick bars	Every N ticks. ★ **N = total ticks ÷ the number of minute bars** (so the counts match)
Returns	The difference between closes, in pips
Kurtosis	Excess kurtosis (0 if normal)
Autocorrelation	Lag 1 of |return|
Hour	UTC, counted at the bar's **closing** time
Baseline	Synthetic random walk down to the tick (3 draws). ★ **The times and the sequence of ticks are left as measured**
Separator	**What the bar was cut on** (the clock, or the number of ticks)
Instruments and period	USDJPY, EURUSD, GBPUSD, XAUUSD / 2019–2024

Observation 2.1: **★ Matching the counts is the point. Without that, the comparison is only "one is finer than the other".**

Observation 2.2: **N is 86 ticks for USDJPY, 76 for EURUSD, 80 for GBPUSD and 151 for XAUUSD.** The counts are **2,181,459**, **2,181,955**, **2,182,565** and **2,090,977** minute bars against **2,183,483**, **2,184,429**, **2,183,491** and **2,090,836** tick bars.

Observation 2.3: **★ In the baseline the times and the sequence of ticks are left as measured, and only the price movement is replaced. So "when and how many ticks arrived" is identical to the measurement.**

Predictions (written before measuring)

① It comes out symmetric: cut by the clock and the tick count varies; cut by ticks and the duration varies.

② ★★ Tick bars' returns have the smaller kurtosis (closer to normal).

③ ★ Tick bars absorb the hour-of-day bias.

④ ★ The autocorrelation of \|return\| is weaker on tick bars.

⑤ On a synthetic random walk the gap in kurtosis between the two shrinks.

⑥ ★ The size of the move per bar is more uniform on tick bars.

3. What was held constant and what was left to chance

Calculation 3.1 Duration and tick count per bar (median of the four instruments)

Bar type	Duration 10th pct	Duration median	Duration 90th pct	Duration spread	Tic
Minute bars	45.5	57.5	59.6	1.31	
Tick bars	16.5	42.3	115.6	7.04	

Observation 3.2: **Prediction ① held. ★ Minute bars vary by only 1.31 in duration while varying by 10.38 in tick count.**

Observation 3.3: **★ Tick bars are the mirror image: 1.00 in tick count (by construction) and 7.04 in duration.**

Observation 3.4: **★★ So the word "a bar" does not stand for the same amount of anything. One counts time, the other counts events.**

---

4. The fat tail halves with the cut

Calculation 4.1 The shape of the returns (median of the four instruments)

Bar type	Count	Mean |r|	Spread	Excess kurtosis	CV of |r|
Minute bars	2,181,707	1.1360	1.9533	120.2788	1.2799
Tick bars	2,183,487	1.3248	1.9415	61.3744	0.9753

Observation 4.2: **Prediction ② held. ★★ Excess kurtosis falls from 120.2788 to 61.3744, a factor of 1.9598.**

Observation 4.3: **Prediction ⑥ held too.** The CV of \|r\| falls from **1.2799** to **0.9753**. **★ [F018] measured the same quantity with the median duration matched instead, and it runs the same way — a check that a different construction gives the same answer.**

Calculation 4.4 Excess kurtosis by instrument

Instrument	Minute bars	Tick bars	Ratio
USDJPY	219.2766	124.6253	1.7595
EURUSD	114.2092	51.3963	2.2221
GBPUSD	126.3484	71.3525	1.7708
XAUUSD	98.3260	50.9362	1.9304

Observation 4.5: **★ All four run between 1.7595 and 2.2221, so the direction agrees.**

Observation 4.6: **★ Even on tick bars, however, it is 50.9362 to 124.6253 — nowhere near normal (0). What the cut removes is half.**

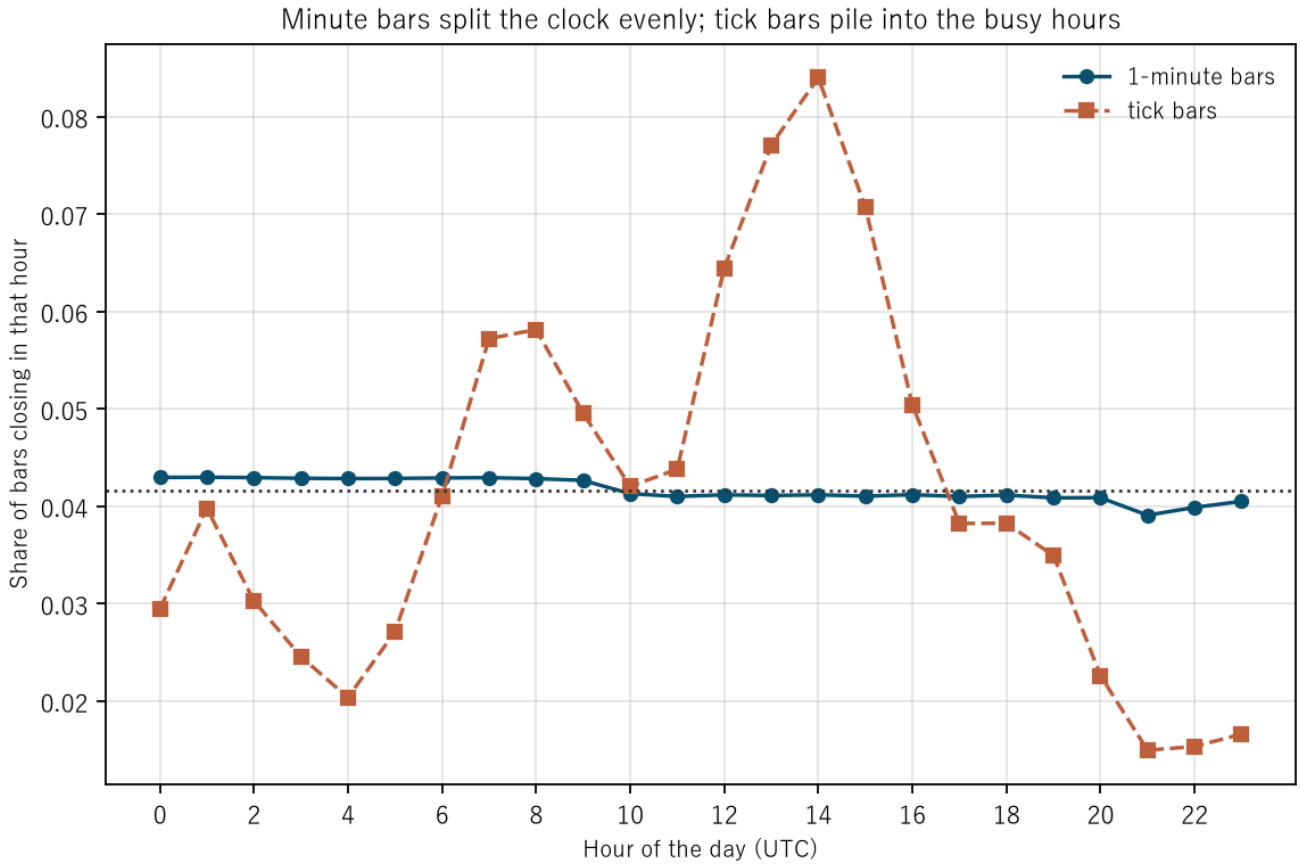


Figure 1: hour of the day (UTC) on the horizontal axis and the share of bars closing in that hour on the vertical. **Circles are minute bars, squares tick bars, and the dotted line is one twenty-fourth.**

5. The clustering halves too

Calculation 5.1 Lag-1 autocorrelation of \|return\|

Instrument	Minute bars	Tick bars	Ratio
USDJPY	0.3280	0.2178	0.6639
EURUSD	0.2858	0.1405	0.4915
GBPUSD	0.2930	0.1758	0.5999
XAUUSD	0.3223	0.1776	0.5511
Median of the four	0.3077	0.1767	0.5743

Observation 5.2: **Prediction ④ held. ★★ The autocorrelation falls from 0.3077 to 0.1767, a factor of 0.5743.**

Observation 5.3: **★ All four run between 0.4915 and 0.6639, so the direction agrees.**

Observation 5.4: **★ Here too, half remains on tick bars. The clustering does not disappear.**

---

6. The hour-of-day bias is not absorbed; it is brought out

Calculation 6.1 Share of bars by hour (median of the four instruments)

Hour (UTC)	Minute bars	Tick bars	Hour (UTC)	Minute bars	Tick bars
0	0.0430	0.0295	12	0.0412	0.0645
1	0.0430	0.0398	13	0.0411	0.0771
2	0.0430	0.0303	14	0.0412	0.0840
3	0.0429	0.0246	15	0.0411	0.0707
4	0.0429	0.0204	16	0.0412	0.0504
5	0.0429	0.0271	17	0.0410	0.0383

Observation 6.2: **Prediction ③ held.** The spread is **1.0647** for minute bars against **6.7105** for tick bars (a factor of **6.3027**).

Observation 6.3: **★★ But "absorb" is the wrong word. Tick bars bring the hour-of-day bias out as a bias in the count.** The most crowded hour is **14**:00 UTC (**0.0840**) and the emptiest is **21**:00 (**0.0150**).

Observation 6.4: **★ That minute bars vary by only 1.0647 follows from splitting the clock evenly. They are not unbiased; they make the bias invisible.**

---

7. ★ The baseline settles it — on a random walk the clock still makes tails and clustering

Calculation 7.1 Replacing only the price movement with a synthetic random walk

Bar type	Measured kurtosis	Random walk's kurtosis	Measured autocorrelation	Randor
Minute bars	120.2788	2.2459	0.3077	
Tick bars	61.3744	−0.0007	0.1767	−0.000

Observation 7.2: ★★ **This is the core of the paper. The random walk's true values are normal (kurtosis 0) and uncorrelated (autocorrelation 0). Cut by tick count, that is exactly what comes out: −0.0007 and −0.0001.**

Observation 7.3: ★★ **Cut the same random walk into minute bars, and an excess kurtosis of 2.2459 and an autocorrelation of 0.1812 appear. Fat tails and volatility clustering out of nothing at all.**

Observation 7.4: ★★ **What makes them is that the number of ticks in a minute differs by hour. Mix minutes with few ticks and minutes with many, and the return distribution moves away from normal and neighbours start to resemble one another.**

Observation 7.5: **Prediction ⑤ held.** The gap in kurtosis (minute minus tick) is **58.9044** measured and **2.2466** on the random walk.

Observation 7.6: ★ **So of the measured kurtosis of 120.2788, what the clock makes is not only 2.2459. What changing the cut removes is 58.9044, and inside that sits the 2.2466 that appears even on a random walk. The rest belongs to the market.**

Observation 7.7: ★ **That a synthetic random walk has no volatility clustering was measured in [F033]. What this paper adds is that a series with no clustering still shows an autocorrelation of**

0.1812 when read on minute bars.

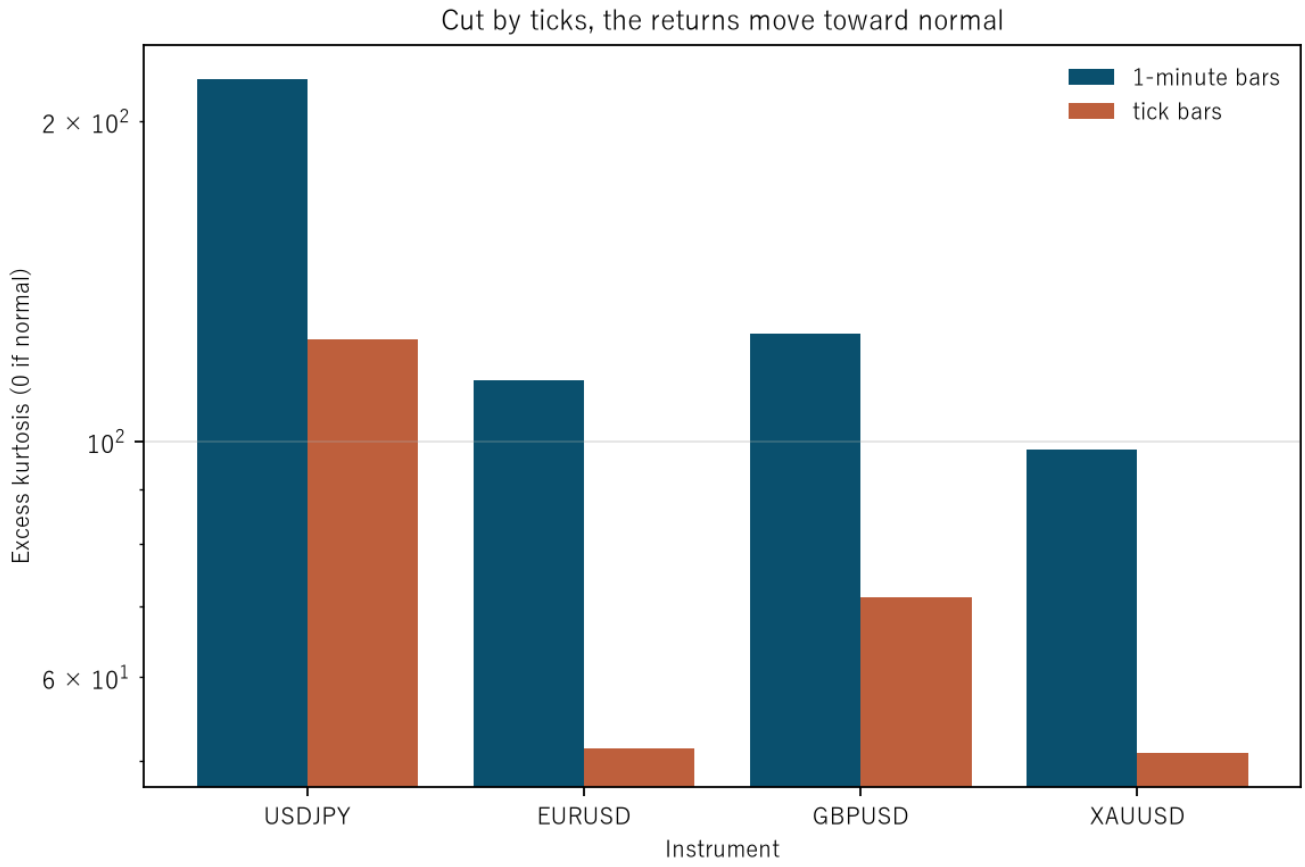


Figure 2: instrument on the horizontal axis and excess kurtosis on the vertical (logarithmic). **Left is minute bars, right tick bars.**

Limitations

1. ⚠️ **Two ways of cutting are fixed**: the clock and the tick count. Volume bars and renko are not included.
2. **One way of matching the counts is fixed**: total ticks ÷ the number of minute bars.
3. **No bar is created for a slot with no ticks.** That this convention moves the answer was measured in [F049].
4. **The spread of the moves overlaps [F018].** What this paper adds is the kurtosis, the autocorrelation and the baseline.
5. **The clock's share and the market's share are not cleanly separated.** All that is said is that the 2.2466 of the random walk sits inside the gap of 58.9044.

6. **No trading is discussed.** Neither performance nor profit and loss is measured.
7. **Four instruments, six years.** Nothing is claimed about other markets or other periods.

Conclusion

To make a bar, something has to be held constant. Minute bars hold time constant; tick bars hold the number of ticks constant.

★ **What is held constant and what is left to chance swap over cleanly.** Minute bars: duration spread **1.31**, tick spread **10.38**. Tick bars: tick spread **1.00**, duration spread **7.04**.

★★ **And the fat tail halves with the cut.** Excess kurtosis goes from **120.2788** to **61.3744** (a factor of **1.9598**), with all four instruments running the same way.

★★ **The volatility clustering halves too.** The autocorrelation goes from **0.3077** to **0.1767** (a factor of **0.5743**).

★★ **The baseline settles it. A synthetic random walk whose true values are normal and uncorrelated shows an excess kurtosis of 2.2459 and an autocorrelation of 0.1812 when cut into minute bars, and returns to −0.0007 and −0.0001 when cut by ticks. Out of nothing at all, the clock makes tails and clustering.**

★ **What makes them is that the number of ticks in a minute differs by hour.** Tick bars run from **0.0840** at **14**:00 UTC to **0.0150** at **21**:00, a spread of **6.7105**, and minute bars show that as **1.0647**.

★ **What the cut removes, though, is half.** Even on tick bars the kurtosis is **50.9362** to **124.6253**, far from normal. **The rest belongs to the market.**

So when saying "minute-bar returns have fat tails and volatility clusters", the clock's share has to be separated out. ★ Even a series whose true value is zero shows 2.2459 and 0.1812 when read on minute bars.

Data and reproduction

Data	Foreign exchange from Dukascopy-derived ticks (from 2019 onward, matching to the millisecond, bid and ask, has been verified). Gold from Dukascopy
Period	2019–2024 (six years)
Instruments	USDJPY, EURUSD, GBPUSD, XAUUSD
A, minute bars	Every 60,000 milliseconds. No bar is created for a slot with no ticks
B, tick bars	Every N ticks. N = total ticks ÷ the number of minute bars, per instrument and year
Returns	The difference between closes, in pips
Kurtosis	Excess kurtosis (0 if normal)
Autocorrelation	Lag 1 of |return|
Hour	UTC, counted at the bar's closing time
Baseline	Synthetic random walk down to the tick (3 draws). The times and spreads are left as measured
Verification	`検算/F050.py` (`--zu` also writes the figures). Every number in the text comes from `検算/F050_output.txt`
Figures	`図/F050_zu1`, `図/F050_zu2` (png, svg, English versions)
Random seeds	Fixed. Re-running gives the same values
Substrate check	`_ツール/audit_bars.py` (bar look-ahead, daylight saving, day boundary). The record is `_土台の検査.txt`

References

- Yamagishi, Y. (2026). Bars Cut by Tick Count Are a Different Clock from Bars Cut by the Clock [F018]. Zenodo. 10.5281/zenodo.22891916
- Yamagishi, Y. (2026). A Synthetic Random Walk Has No Volatility Clustering [F033]. Zenodo. 10.5281/zenodo.22893674
- Yamagishi, Y. (2026). Create Bars for the Empty Slots and the Values Stay Put While the Counts Move [F049]. Zenodo. 10.5281/zenodo.22897116