The Total Fertility Rate Is the Sum of That Year's Age-Specific Birth Rates, Not the Number of Children a Woman Has in Her Lifetime — In All 20 Years from 1981 to 2000, the Year's Value Was 0.041–0.317 Lower Than the Completed Fertility of Women Born 30 Years Earlier, and in 23 of the Years from 1995 to 2019 It Was Lower Than That of Every Birth Cohort Counted Through Age 49 (Minimum 1.429). The Faster the Mean Age at Birth Rose, the Larger the Gap [M102]

That year's total fertility rate ≠ completed fertility of a birth cohort ── What separates them is whether ages are summed within the same year or within the same birth cohort

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Abstract

Using the age-specific birth rates by birth cohort × single year of age from the Ministry of Health, Labour and Welfare's Vital Statistics special report "Statistics on Births" (FY2021; women born 1932–2004, ages 15–49) and the yearly total fertility rate in the Vital Statistics (Table 4.1), this paper looked at what the total fertility rate counts. No new mathematical theorem and no new law are claimed.

First, **from one and the same table, summing ages within the same year gives the total fertility rate (period sum), and summing ages within the same birth cohort gives the number of children women of that cohort had by age 49 (completed fertility).** Values built by year from the same table matched the total fertility rate in Table 4.1 within 0.005 for all 39 years from 1981 to 2019 (Section 3).

Second, **completed fertility for the cohorts born 1932–1970, counted through age 49, ranges from 1.429 (born 1967) to 2.105 (born 1945); cohorts born 1932–1957 are mostly around 2, then it fell to 1.846 for those born 1960 and 1.466 for those born 1970** (Section 4).

Third, this is the core of the paper. **In all 20 years from 1981 to 2000, that year's total fertility rate was lower than the completed fertility of women born 30 years earlier (gap −0.317 to −0.041). In 23 of the 25 years from 1995 to 2019 (all except 2015 and 2016), it was lower than the completed fertility of every cohort counted through 49, with the lowest being 1.260 in 2005.** "That year's sum" lined up in the same year is not a count of any woman's lifetime (Section 5).

Fourth, **the (period) mean age at birth rose from 27.34 years in 1981 to 29.15 in 2000 and 31.01 in 2019, rising by 0.079–0.150 years per year in 1982–1996. The faster the mean age rose, the more negative the gap between that year's value and the completed fertility of the cohort born 30 years earlier (correlation −0.608, 19 years 1982–2000)** (Section 5).

Fifth, **the total fertility rate in 1966 (hinoeuma) was 1.58, lower than 2.14 in 1965 and 2.23 in 1967. In the birth-cohort table this one year appears as a diagonal line: for all 16 cohorts born 1932–1947, the rate at the age corresponding to 1966 was 0.670–0.786 times the average of the neighboring ages. Even so, completed fertility for those cohorts was 1.807–2.105** (Section 6).

Sixth, what separates them is **whether ages are summed within the same year or within the same birth cohort**. "The total fertility rate was 1.26" is the sum of that year's age-specific birth rates, not a count showing that women had 1.26 children on average in their lifetime.

Scope note: **This paper claims no new mathematical theorem and no new law** — age-specific birth rates and total fertility rates are in public tables, and this paper only summed the same table by year and by birth cohort and computed gaps, mean ages, and a correlation. **This paper is a structural interpretation of the shape of numbers in public aggregate data; it is not medical advice, not a diagnosis, and not a treatment recommendation. The author is not a physician. Decisions about individual health should be discussed with a physician or a medical institution. This is a preprint that has not been peer reviewed. It does not judge fertility as high or low, good or bad, does not discuss the reasons (marriage, work, housing, systems, etc.), and does not evaluate policy.** The correlation between the rise in mean age and the gap is the direction of the numbers lined up; which moved which cannot be said. No formula converting that year's value into lifetime fertility is used. The special report's table runs to 2019; for 2020 and later only Table 4.1's values are used. Why completed fertility for those born in 1966 and 1967 (1.727 and 1.429) departs from neighboring cohorts cannot be known from these tables. **About the data**: the Vital Statistics special report "Statistics on Births" and the Vital Statistics (Ministry of Health, Labour and Welfare) were taken from e-Stat and used under the Government of Japan Standard Terms of Use (Version 2.0); the sums by year and by birth cohort, mean ages, gaps, ratios, and correlation in this paper were processed from them. What this paper adds: **lining up the total fertility rate and completed fertility by birth cohort derived from one and the same table, and showing that in every year from 1981 to 2000 that year's value was lower than the completed fertility of the cohort born 30 years earlier, and that in 23 of the years from 1995 to 2019 it was lower than that of every completed cohort**.

1. Notation and question

- **Age-specific birth rate**: births per 1,000 women of that age.
- **Period sum (total fertility rate)**: the sum of age-specific birth rates at ages 15–49 in the same year ÷ 1000.

- **Completed fertility**: the sum of age-specific birth rates at ages 15–49 for women of the same birth cohort ÷ 1000 (complete only for cohorts counted through age 49).
- **Mean age**: Σ (age × rate) ÷ Σ rate.

Question: Is the total fertility rate the same as the number of children a woman has in her lifetime? If not, by how much, and in what kind of years, does it differ?

Observation 1.1: M003 showed that life expectancy is built by lining up that year's death rates and is not the number of years anyone actually lives. The total fertility rate is likewise built by lining up that year's age-specific birth rates. This paper applies the same kind of question to births.

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2. Preview and design

Written honestly: the design was decided after looking at the tables.

	Content
Tables	Vital Statistics special report "Statistics on Births" (FY2021), Table 6-2 (age-specific birth rates by birth cohort) and Table 7 (cumulative birth rates by birth cohort); Vital Statistics 2024, Volume 1, Table 4.1 (yearly total fertility rate)
Noticed in the preview	Table 6-2 gives rates by birth cohort × single year of age; regrouped diagonally by year, they give the total fertility rate, which can be checked against Table 4.1
Noticed in the preview (2)	The rate at age 34 (1966) for those born 1932 was lower than at neighboring ages; the hinoeuma year appeared diagonally
Noticed in the preview (3)	Completed fertility for those born 1966 and 1967 (1.727 and 1.429) departed from neighboring cohorts. The reason is not known from these tables, so it is stated as is
Comparison	Completed fertility by cohort, the gap between that year's value and the completed fertility of the cohort born 30 years earlier, the rise in mean age, and the diagonal line of the hinoeuma year

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3. Data and checks

Calculation 3.1 The 3 retrieved files were fixed by SHA-256, and the following were checked.

- Tables 6-2 and 7: birth cohorts are the 73 years 1932–2004, and ages are the 35 years 15–49. The notes say the total is computed over ages 15–49 (up to 2019 for cohorts that have not reached 49).
- Summing Table 6-2 along a birth cohort matches the table's total within 0.015 (73 cohorts). The cumulative birth rate at age 49 in Table 7 matches Table 7's total. The totals of the two tables are the same except for one, those born 2001 (0.01 and 0.00) (I first assumed they were all the same and the check failed).
- Years that can be built from the same table are the 39 years 1981–2019 in which ages 15–49 are all available, and their sums match the total fertility rate in Table 4.1 within 0.005 (largest difference 0.0048).

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4. Completed fertility by birth cohort

Calculation 4.1 Completed fertility and mean age at birth for cohorts counted through age 49

Birth cohort	Completed fertility	Mean age at birth
1932	2.039	26.81
1945	2.105	26.84
1950	2.027	27.08
1955	1.976	27.65
1960	1.846	28.19
1965	1.602	28.83
1967	1.429	29.11
1970	1.466	29.47

Observation 4.2: **Completed fertility for the cohorts born 1932–1970 ranges from 1.429 (born 1967) to 2.105 (born 1945). Those born 1932–1957 are mostly around 2, and it fell from the cohort born 1960. Mean age at birth rose from 26.81 years (born 1932) to 29.47 (born 1970).**

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5. The core — that year's value and the completed fertility of women born 30 years earlier

Calculation 5.1 That year's total fertility rate (period sum) and the completed fertility of the cohort born 30 years earlier

Year	That year's value	Cohort born 30 years earlier	Completed fertility	Gap	Rise in mean age per year
1982	1.770	1952	2.006	−0.235	+0.099
1987	1.690	1957	1.971	−0.281	+0.150
1989	1.572	1959	1.889	−0.317	+0.124
1993	1.458	1963	1.708	−0.250	+0.101
1997	1.388	1967	1.429	−0.041	+0.056
2000	1.359	1970	1.466	−0.108	+0.010

Observation 5.2: **In all 20 years from 1981 to 2000, that year's value was lower than the completed fertility of women born 30 years earlier (gap −0.317 to −0.041). The correlation between the yearly rise in mean age and the gap is −0.608 (19 years 1982–2000): the faster the rise, the more negative the gap.** The period mean age rose from 27.34 years in 1981 to 29.15 in 2000 and 31.01 in 2019, rising by 0.079–0.150 years per year in 1982–1996.

Observation 5.3: **In 23 of the 25 years from 1995 to 2019, that year's value was lower than the completed fertility of every cohort counted through 49 (minimum 1.429). The 2 exceptions are 2015 (1.450) and 2016 (1.441). The lowest is 1.260 in 2005.** In Table 4.1 it is 1.15 in 2024.

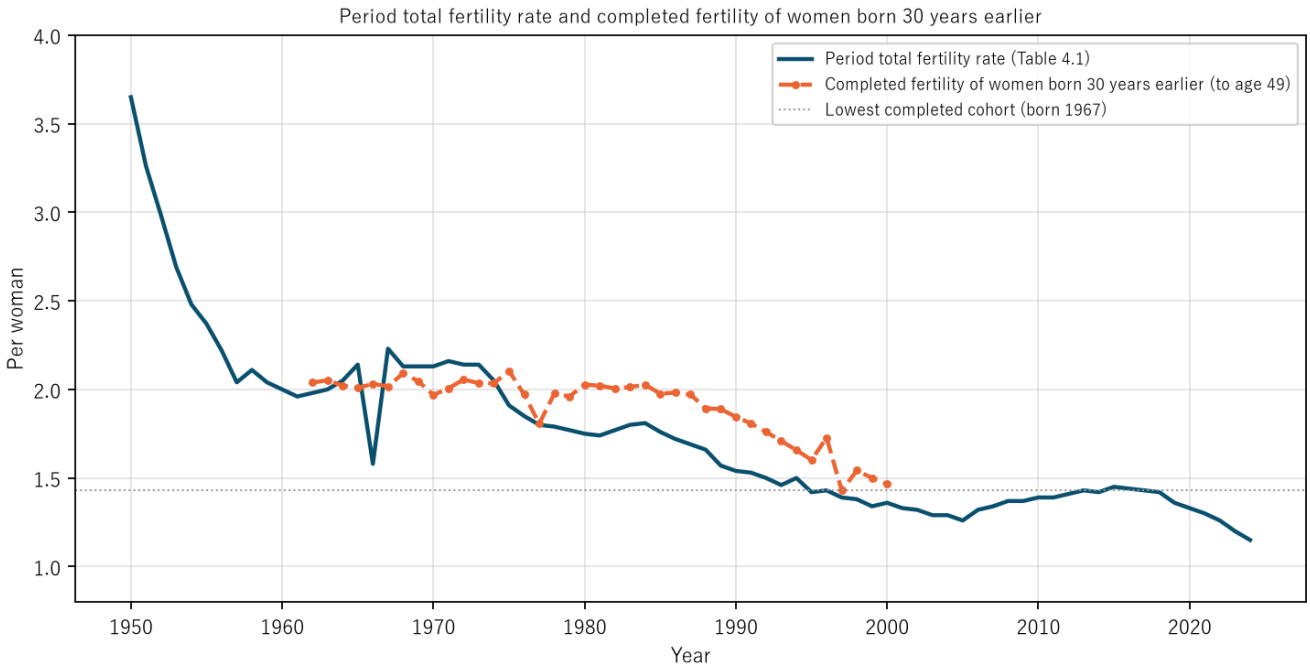


Figure 1: Navy is that year's total fertility rate (Table 4.1); the orange dashed line is the completed fertility of women born 30 years earlier (cohorts born 1932–1970, counted through 49); the dotted line is the lowest completed cohort (born 1967, 1.429).

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6. The hinoeuma year appears as a diagonal line

Observation 6.1: **The total fertility rate in 1966 was 1.58, lower than 2.14 in 1965 and 2.23 in 1967. In the birth-cohort table, this year falls at a different age for each cohort (age 34 for those born 1932, age 19 for those born 1947), forming a diagonal line. For all 16 cohorts born 1932–1947 whose neighboring-age average is 10 or more, the rate at the age corresponding to 1966 was 0.670–0.786 times the average of the neighboring ages. Even so, completed fertility for those born 1932–1951 is 1.807–2.105.** A single year's event can move that year's sum greatly, yet in completed fertility by birth cohort it is diluted as the share of one age.

Completed fertility for those born 1966 and 1967 is 1.727 and 1.429, departing from those born 1965 (1.602) and 1968 (1.542). The reason is not known from these tables.

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7. Limitations

1. **It does not judge fertility as high or low, good or bad, does not discuss the reasons, and does not evaluate policy.**
2. The correlation between the rise in mean age and the gap is the direction of the numbers lined up; which moved which cannot be said. No formula converting that year's value into lifetime fertility is used.
3. The birth-cohort table runs to 2019. Cohorts born 1971 and later have not yet reached 49 (their lifetime counts are not complete).
4. Birth order is not separated. Why completed fertility for those born 1966 and 1967 departs is not known.

Conclusion

The total fertility rate was the sum of that year's age-specific birth rates, not the number of children a woman has in her lifetime. Summing the same table by birth cohort, in every year from 1981 to 2000 that year's value was lower than the completed fertility of women born 30 years earlier, and in 23 of the years from 1995 to 2019 it was lower than that of every completed cohort. The faster the mean age at birth rose, the larger the gap.

What separates them is one thing — whether ages are summed within the same year or within the same birth cohort.

When you hear that "the total fertility rate was X," **remember too that it is that year's sum, a different number from completed fertility by birth cohort.**

Data and reproduction

Data	Vital Statistics special report "Statistics on Births" (FY2021), Tables 6-2 and 7; Vital Statistics 2024, Volume 1, Table 4.1 (Ministry of Health, Labour and Welfare, e-Stat) (retrieved 26 September 2026)
Terms of use	e-Stat follows the Government of Japan Standard Terms of Use (Version 2.0). The sums by year and by birth cohort, mean ages, gaps, ratios, and correlation in this paper were processed from it and are not numbers produced by the government
Coverage	Japan, women born 1932–2004 at ages 15–49 (to 2019); total fertility rate 1947–2024
Verification	`検算/M102.py` (`--zu` also draws the figure). Every number in the text is taken from `検算/M102_output.txt`
Raw data	`data/` holds the retrieved files and `SOURCES.md` (URLs, retrieval date, SHA-256, terms of use, definitions and how the calculations were set up)
Figures	`図/M102_zu1` (png, svg, English version)

References

- Ministry of Health, Labour and Welfare. Vital Statistics special report FY2021, Statistics on Births, Tables 6-2 and 7 (in Japanese). e-Stat (accessed 26 September 2026). https://www.e-stat.go.jp/
- Ministry of Health, Labour and Welfare. Vital Statistics 2024, final figures, Volume 1, births, Table 4.1 (in Japanese). e-Stat (accessed 26 September 2026). https://www.e-stat.go.jp/
- Yamagishi, Y. (2026). Life Expectancy Is Not the Number of Years Anyone Actually Lives [M003]. Zenodo. https://doi.org/10.5281/zenodo.22963485